AD 405 831

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA

DAVIDSON LABORATORY

SURGING MOTION AND BROACHING TENDENCIES IN A SEVERE IRREGULAR SEA

Dr. Otto Grim

STEVENS INSTITUTE OF TECHNOLOGY
CASTLE POINT STATION
HOBOKEN NEW JERSEY

R-929 Nov 1962

SURGING MOTION AND BROACHING TENDENCIES IN A SEVERE IRREGULAR SEA

Dr. Otto Grim
Institute für Schiffbau
Hamburg, Germany

Report No. 929
November 1962
DL Project No. 004

Sponsored by
Office of Naval Research
under Contract Nonr 263(10)

Davidson Laboratory
Stevens Institute of Technology
Hoboken, New Jersey

ABSTRACT

Nonlinearities are very significant for the surging motion of a ship in a following sea. Results are presently available only for the surging motion in regular following waves. The law of superposition cannot be used to treat the nonlinear surging motion in an irregular sea. Therefore, it is not possible to extend the results obtained in regular waves to predict the behavior in an irregular sea.

A method to overcome these difficulties to investigate nonlinear surging motion in an irregular sea is presented in Part 1 of this report. Using this method, results are obtained for nonlinear surging motion; in particular, solutions are given for the acceleration of a ship to an increased speed and for the subsequent run at that increased speed.

Based on the results obtained in Part 1, the problem of broaching is discussed in Part 2. A connection between broaching and nonlinear surging motion was presumed in previous discussions about broaching. This connection is confirmed and, furthermore, nonlinear surging motion is found to be the most important pre-condition for broaching.

TABLE OF CONTENTS

SURGING MOTION OF A SHIP IN AN IRREGULAR FOLLOWING SEA

INTRODUCTION

To compute the surging motion of a ship in a head sea, a linearized equation of motion can be assumed. This leads to a straightforward solution in regular waves. The surging motion in an irregular head sea can be treated as a random process and solved by the method of St. Denis and Pierson.[1]

The assumption of linearization may not, however, be valid for surging motion in a following sea, since nonlinearities become significant in this case. This fact was demonstrated in refs. 2 and 3 where it was noted that in a certain range of propeller thrust, the oscillatory surging motion vanishes in regular following seas and the ship is forced to run at the same speed as the wave. This behavior is very striking and is believed to be connected in some way with many known cases of ships broaching in following seas.

Because this striking behavior in a regular following wave is due to nonlinearities, no conclusion about the analogous behavior in an irregular following sea is possible. A treatment of the surging motion in an irregular following sea must include the influence of these nonlinearities and great difficulties arise from this requirement. No scientific treatment of this sort is known to the author.

This paper attempts to overcome these difficulties. No complete solution of the problem is attained, but some answers are found -- particularly to the question of whether the ship can be accelerated to a greater extent than may be predicted by a linear equation of motion.

THE EQUATION OF SURGING MOTIONS AND THE ROLE OF NONLINEARITIES

A two-dimensional irregular sea may be represented by:

$$h(x,t) = \sum_{n=0}^{N} \sqrt{r(\omega_n)\,\Delta\omega_n} \quad \cos\left[\frac{\omega_n^2}{g}x + \omega_n t + \sigma_n\right]$$
$$\text{or } = \int_0^\infty \cos\left[\frac{\omega^2}{g}x + \omega t + \sigma(\omega)\right] \sqrt{r(\omega)\cdot d\omega} \tag{1}$$

The force in the longitudinal direction produced by a regular longitudinal wave (the exciting surging force) can be approximated by using the Froude-Kryloff hypothesis. The ratio of this force divided by the wave amplitude can be represented by:

$$\frac{F}{h} = \frac{2\pi}{\lambda}\cdot\nabla\cdot f\left(\frac{\lambda}{L}\right) = \frac{\omega^2}{g}\ \nabla\cdot f\left(\frac{\omega^2 L}{g}\right) \tag{2}$$

where $f\left(\frac{\lambda}{L}\right) = f\left(\frac{\omega^2 L}{g}\right)$ and denotes the forcing function of the exciting surging force. This function depends on the form of the ship and on the ratio of wave length to ship length. For the numerical computation carried out and represented in this paper, the following is used as a possible function (Fig. 1):

$$f\left(\frac{\lambda}{L}\right) = 3\ \frac{\sin\left(\frac{\omega^2 L}{2g}\right) - \left(\frac{\omega^2 L}{2g}\right)\cos\left(\frac{\omega^2 L}{2g}\right)}{\left(\frac{\omega^2 L}{2g}\right)^3} \tag{3}$$

In the same manner, the surging force on a ship in a two-dimensional irregular sea running longitudinally becomes:

$$F = \int_0^\infty \cos\left[\frac{\omega^2}{g}x + \omega t + \sigma(\omega)\right]\sqrt{\frac{\omega^4}{g^2}\ \nabla^2\ f^2\ \left(\frac{\lambda}{L}\right)\ r(\omega)\ d\omega} \tag{4}$$

It is convenient to divide this force by the mass of the ship: $m = \frac{\nabla}{g}$. The result has the dimensions of acceleration:

$$\frac{F}{m} = \int_0^\infty \cos\left[\frac{\omega^2}{g} x + \omega t + \sigma(\omega)\right] \sqrt{\omega^4 f^2(\tfrac{\lambda}{L}) r(\omega) d\omega} \qquad (5)$$

If we assume that the ship is moving at a constant mean velocity V, where the propeller thrust is equal to the calm water resistance, we may choose our coordinate system for x_o, the surging motion, where the origin is also moving at velocity V. The equation for surging motion can then be written as:

$$(m+M_{xx})\ \ddot{x}_o + N\dot{x}_o = m \int_0^\infty \cos\left[\frac{\omega^2}{g} x_o + \omega_e t + \sigma(\omega)\right] \sqrt{\omega^4 f^2(\tfrac{\lambda}{L}) r(\omega)\ d\omega} \qquad (6)$$

The added mass may be small and will be neglected for the purpose of this paper. The damping force $N\dot{x}_o$ will be insignificant in most cases.

Equation 6 is nonlinear due to the term x_o on the right-hand side. This nonlinearity is insignificant in the cases of a ship at zero speed or running in head seas, but may be significant for a ship running in following seas.

Summarizing, for the ship running in head seas or for the ship at zero speed, the terms x_o and M_{xx} can be neglected. Then eq. 6 becomes:

$$\ddot{x}_o + \frac{N}{m}\dot{x}_o = \int_0^\infty \cos\left[\omega\left(1+\frac{\omega V}{g}\right)t + \sigma(\omega)\right] \cdot \sqrt{\omega^4 f^2(\tfrac{\lambda}{L})\ r(\omega)\ d\omega} \qquad (7)$$

and the spectrum for the motion, that is for x_o, $\dot{x}_o$ or $\ddot{x}_o$, can be determined. For instance, $\dot{x}_o$ will be:

$$\dot{x}_o = \int_0^\infty \cos\left[\omega(1+\frac{\omega V}{g})\, t + \sigma(\omega)\right] \sqrt{\frac{\omega^4\, f^2(\frac{\lambda}{L})\; r(\omega)}{(\frac{N}{m})^2 + \omega^2\,(1+\frac{\omega V}{g})^2}}\; d\omega \qquad (8)$$

If the same assumptions were used for the ship in a following sea, eq. 7 would be replaced by:

$$\ddot{x}_o + \frac{N}{m}\dot{x}_o = \int_0^\infty \cos\left[\omega(1-\frac{\omega V}{g})\, t + \sigma(\omega)\right] \sqrt{\omega^4\, f^2\,(\frac{\lambda}{L})\; r(\omega)\, d\omega} \qquad (9)$$

and instead of eq. 8, the following surging velocity would be found:

$$\dot{x}_o = \int_0^\infty \cos\left[\omega(1-\frac{\omega V}{g})\, t + \sigma(\omega)\right] \sqrt{\frac{\omega^4\, f^2\,(\frac{\lambda}{L})\; r(\omega)}{(\frac{N}{m})^2 + \omega^2\,(1-\frac{\omega V}{g})^2}}\; d\omega \qquad (10)$$

Because $(1 - \frac{\omega V}{g})$ equals 0 for a certain circular frequency ω and because $\frac{N}{m}$ will be a small value, eq. 10 would produce a large velocity $\dot{x}_o$. Moreover, the displacement x_o computed from equation 9 or equation 10 would become infinite. Therefore, it is not permissible to neglect x_o on the right-hand side of equation 6 for a ship in a following sea.

Reference 2 shows that the nonlinearity represented by the term x_o on the right-hand side of eq. 6 is significant in the case of the ship in a regular following sea and is responsible for the striking behavior mentioned in the Introduction. Unfortunately, a scientific treatment of this behavior in irregular following seas is not possible by the method of St. Denis and Pierson.[1] Therefore, another method is proposed which does not result in a complete solution but does provide answers to certain basic questions about surging motions in irregular following seas.

THE IMPULSE SPECTRUM

The way to such answers is opened by introducing an impulse spectrum which will be described below.

Equation 6 may be sufficiently correct to satisfy the aims of this paper provided that a means of solving this equation can be found. The equation is nonlinear, however, and it is not possible to obtain a solution in the conventional manner.

A different approach will now be tried. Let time be stopped at an arbitrary point, t, and let the succeeding time be represented by τ . Neglecting M_{xx}, eq. 6 is now written as

$$\ddot{x}_o + \frac{N}{m}\dot{x}_o = \int_0^\infty \cos\left[\frac{\omega^2}{g}x_o + \omega_e(t+\tau) + \sigma(\omega)\right]\sqrt{\omega^4 f^2(\tfrac{\lambda}{L})\,r(\omega)\,d\omega} \quad (11)$$

Equation 11 can be written for a ship in a head sea as follows:

$$\ddot{x}_o + \frac{N}{m}\dot{x}_o = \int_0^\infty \cos\left[\frac{\omega^2}{g}x_o + \omega(1+\frac{\omega V}{g})\,(t+\tau) + \sigma(\omega)\right]\sqrt{\omega^4 f^2(\tfrac{\lambda}{L})\,r(\omega)\,d\omega} \quad (12)$$

and for a ship in a following sea:

$$\ddot{x}_o + \frac{N}{m}\dot{x}_o = \int_0^\infty \cos\left[\frac{\omega^2}{g}x_o - \omega(1-\frac{\omega V}{g})(t+\tau) + \sigma(\omega)\right]\sqrt{\omega^4 f^2(\tfrac{\lambda}{L})\,r(\omega)\,d\omega} \quad (13)$$

To illustrate, assume that in a model test, the model initially is attached to a carriage moving at a constant speed, V, at which the mean propeller thrust equals the mean resistance (see remarks preceding eq. 6). Then, at an arbitrary time t, and $\tau = 0$, the model is released from its attachment to the carriage. For the free model, time is now

represented by τ and t is considered as a constant. Obviously it is not possible to make the same assumption for a full scale ship. But this will not exclude the use of the same method for the ship, because in an irregular sea a severe sea can follow a long interval of relatively moderate seas.

Using this concept, x_o, $\dot{x}_o$, and $\ddot{x}_o$ in eqs. 11, 12, or 13 will represent functions of τ with certain conditions at $\tau = 0$. Obviously it is not possible to solve eqs. 11, 12 and 13 in this form for the reason given in connection with eq. 6. Next, both sides of these equations are integrated over τ from $\tau = 0$ to $\tau = T$. Equation 13 then becomes

$$\int_0^T \left[\ddot{x}_o + \frac{N}{m}\,\dot{x}_o\right] d\tau = \int_0^T \int_0^\infty \cos\left[\frac{\omega^2}{g}\;x_o - \omega\,(1 - \frac{\omega V}{g})\;(t+\tau) + \sigma(\omega)\right] \sqrt{\omega^4\;f^2(\frac{\lambda}{L})\,r(\omega)\;d\omega}\;d\tau \tag{14}$$

The left-hand side of eq. 14 becomes

$$\dot{x}_o\;(t+T) - \dot{x}_o(t) + \frac{N}{m}\left[x_o(t+T) - x_o(t)\right] \tag{15}$$

In general, the right-hand side of eq. 14 cannot be integrated and a complete solution is not possible. But the following procedure will now be tried. If an arbitrary surge motion, x_o, is assumed, the right-hand side of eq. 14 can be integrated with respect to τ. The result of this integration can then be compared with exp. 15 to see if the assumed x_o will satisfy that equation. This is the most important part of the proposed method and the spectrum obtained as a result of integrating the right-hand side of eq.14 with respect to τ is called an impulse spectrum.

The most interesting question about surging motion in a following sea is whether the ship can be accelerated by the waves and maintain a speed higher than its mean speed for an extended time. To examine this, first assume an

accelerated motion, x_o, for computing the impulse spectrum and then test this assumption to see if such a motion satisfies eq. 14.

Now for x_o, assume an accelerated motion with a constant acceleration, b.

$$x_o = \frac{b}{2}\tau^2 \tag{16}$$

Then, the right-hand side of eq. 14 becomes:

$$\mathrm{Re}\int_0^\infty e^{i\left[-\omega\left(1-\frac{\omega V}{g}\right)t+\sigma(\omega)\right]}\sqrt{\omega^4 f^2\left(\frac{\lambda}{L}\right)r(\omega)\,d\omega}\int_0^T e^{i\left[-\omega\left(1-\frac{\omega V}{g}\right)\tau+\frac{\omega^2}{g}\frac{b}{2}\tau^2\right]}d\tau \tag{17}$$

Now the integration over τ can be performed:

$$\int_0^T e^{i\left[-\omega\left(1-\frac{\omega V}{g}\right)\tau+\frac{\omega^2}{g}\frac{b}{2}\tau^2\right]}d\tau \tag{18}$$

If b = 0, expression 18 becomes:

$$e^{-i\frac{\omega T}{2}\left(1-\frac{\omega V}{g}\right)}\frac{\sin\left[\frac{\omega T}{2}\left(1-\frac{\omega V}{g}\right)\right]}{\frac{\omega}{2}\left(1-\frac{\omega V}{g}\right)} \tag{19a}$$

If $b \neq 0$, expression 18 becomes:

$$\frac{e^{-i\left(1-\frac{\omega V}{g}\right)^2\frac{g}{2b}}}{\omega\sqrt{\frac{b}{2g}}}\int_{-\left(1-\frac{\omega V}{g}\right)\sqrt{\frac{g}{2b}}}^{\left[-\left(1-\frac{\omega V}{g}\right)\sqrt{\frac{g}{2b}}+\omega T\sqrt{\frac{b}{2g}}\right]} e^{iu^2}du \tag{19b}$$

The integral in exp. 19b is known as a Fresnel Integral and is tabulated in ref. 4. If the computations are to be performed on an electronic digital computer, it will be more convenient to integrate after e^{iu^2} is expanded in a series.

The exponential function in exps. 19a and 19b can be included in a new phase, $\sigma(\omega)$, of the resulting representation of the impulse. Without this exponential function, exp.19b is T times a dimensionless complex function of (ωT), $(1 - \frac{\omega V}{g})$ and $(\frac{b}{g})$:

$$\frac{1}{\omega\sqrt{\frac{b}{2g}}} \int_{-(1 - \frac{\omega V}{g})\sqrt{\frac{g}{2b}}}^{\left[-(1-\frac{\omega V}{g})\sqrt{\frac{g}{2b}} + \omega T\sqrt{\frac{b}{2g}}\right]} e^{iu^2} du = T \left\{ I\left[\omega T, (1-\frac{\omega V}{g}), \frac{b}{g}\right] + iY\left[\omega T, (1-\frac{\omega V}{g}), \frac{b}{g}\right] \right\} \tag{20}$$

Then exp. 17 can be replaced by

$$\int_0^\infty \cos\left[\omega(1 - \frac{\omega V}{g}) t + \sigma(\omega)\right] \sqrt{\omega^4 f^2(\frac{\lambda}{L}) \; T^2(I^2+Y^2) \; r(\omega) d\omega} \tag{21}$$

The phase σ in exp. 21 is not the same as that in exp.17 but this is not important since in each case σ is a random value. The spectrum

$$\omega^4 f^2(\frac{\lambda}{L}) \; T^2 \; (I^2+Y^2) \; r(\omega) \tag{22}$$

may be called the impulse spectrum for the surging motion.

The impulse (exp. 21) is represented as an arbitrary superposition of a finite or infinite number of harmonic components in the same manner as the irregular seaway itself. It is dependent on

1. the seaway spectrum, r
2. the forcing function, $f(\frac{\lambda}{L})$
3. the time for which the impulse is computed, T
4. the assumed surge motion, x_o

Now eq. 14 must be satisfied. Using the assumed motion of eq. 16 on the left-hand side of eq. 14 as represented in exp. 15:

$$bT+\frac{N}{m}\left(\frac{b}{2}T^2\right) = \int_0^\infty \cos\left[\omega\left(1-\frac{\omega V}{g}\right)t+\sigma(\omega)\right]\sqrt{\omega^4 f^2\left(\frac{\lambda}{L}\right)T^2(I^2+Y^2)\ r(\omega)\,d\omega} \tag{23}$$

This equation is satisfied only for a unique value of b and it can be used to estimate this acceleration, b, as a function of the time interval, T. The influence of the nonlinearity due to the term x_0 on the right-hand side of eq. 14 will be included in this estimate.

APPLICATION TO THE BEHAVIOR OF A SHIP IN A REGULAR FOLLOWING SEA

To demonstrate what can be expected of the proposed method, first the case in regular following waves is treated. For this case, a result found by a more exact method is known.[2] For this special case, eq. 23 becomes:

$$bT+\frac{N}{m}\frac{b}{2}T^2 = \cos\left[\omega\left(1-\frac{\omega V}{g}\right)t+\sigma(\omega)\right]\ \omega^2 f\ \left(\frac{\lambda}{L}\right)\ T\sqrt{I^2+Y^2}\ \bar{h} \tag{24}$$

because in this case $\sqrt{r(\omega)\,d\omega}$ must be replaced by the wave amplitude $\bar{h}$.

It is convenient to replace $\left[\omega^2 f\ \left(\frac{\lambda}{L}\right)\bar{h}\right]$ by $\bar{b}$. This value $\bar{b}$ represents the amplitude of the acceleration which can be produced by the wave. This can be seen from eqs. 2 and 5. Then eq. 24 is rewritten:

$$bT+\frac{N}{m}\frac{b}{2}T^2 = \bar{b}\ T\sqrt{I^2+Y^2}\quad \cos\left[\omega\left(1-\frac{\omega V}{g}\right)t+\sigma(\omega)\right] \tag{25}$$

Remember the meaning of b: it represents an average acceleration over a time interval T which begins at the time t.

We are most interested in the maximum value of the impulse. For this maximum value, the harmonic function in eq. 25 is replaced by 1. This harmonic function only serves to show that the impulse depends on the time t at which the model is released from the carriage and that this dependence is harmonic. For this most interesting case, eq. 25 becomes:

$$bT+\frac{N}{m}\frac{b}{2}T^2 = \bar{b}T\sqrt{I^2+Y^2} \tag{26}$$

Remember that I and Y are functions of ωT, $(1-\frac{\omega V}{g})$, $\frac{b}{g}$. Now for the regular waves, given by ω and $\bar{b}$, and for a mean velocity V, a time interval T is chosen and the maximum value of the average acceleration b for this time interval or the increment of velocity bT is determined from eq. 26. This computation is carried out for different time intervals T and the results are presented in Fig. 2.

In Fig. 2 the increment of velocity produced by the waves in the time interval T is plotted versus T. This diagram shows that for small values of T, the increment of velocity does not depend on the velocity V. In this range the average acceleration b equals the maximum possible acceleration $\bar{b}$. But for larger time intervals T the average acceleration b is smaller than $\bar{b}$ because the location of the model relative to the wave is changed within this interval T. The influence of the velocity V on the increment of velocity is very significant. If the velocity V equals the wave velocity $C=\frac{g}{\omega}$, the parameter on the curves equals 0 and the increment of velocity is relatively small. But the higher velocity will continue for a relatively long time. If the velocity V is smaller than the wave velocity, the parameter on the curves increases and the increment of velocity also increases. But if the velocity V becomes too small, a very

striking change can be seen. Then the increment of velocity remains small, and furthermore, this increment exists for only a very short time. It can be concluded from Fig. 2 that for values of the parameter $\left[(1-\frac{\omega V}{g})\sqrt{\frac{g}{\pi \bar{b}}} \right]$ smaller than 1.1, the model will be accelerated to such an extent that it will attain the wave velocity and run with the wave. But for parameters $\left[(1-\frac{\omega V}{g})\sqrt{\frac{g}{\pi \bar{b}}} \right]$ larger than 1.1, the increment of velocity remains small and the model can never maintain the wave velocity.

Nearly the same result is obtained in ref. 2. Quantitatively the agreement is not perfect. But a perfect agreement could not be expected because the question is not exactly the same as in ref. 2. Reference 2 poses the question as to the condition under which a wave will force a ship to run at the wave velocity. On the other hand, the method developed here is used to find what maximum increment of velocity can be imposed on a ship by a wave in a given time interval. In spite of this difference, comparison of the results shows that the method developed in this report is reasonable and can be used to obtain useful answers for a model or a ship in regular following waves.

Of course, the increment of velocity may be very small and may even be negative, depending upon the time t at which the model is released. But we are primarily interested in the largest possible increment of velocity and therefore have replaced the harmonic function in eq. 25 with a value of unity.

Strictly speaking, due to the assumption of eq. 16, Fig. 2 should be used only in the region where the acceleration is constant (close to the line $b = \bar{b}$). However, a good approximation is obtained if each curve's use is restricted to the range below the point of maximum increment of velocity.

It should be remembered that nonlinearities have an important effect on these results. An accelerated ship remains on the slope of a wave which produces the acceleration a longer interval of time than does a non-accelerated ship. This effect is included in the computation only if the nonlinearities are taken into account.

APPLICATION OF THE PROPOSED METHOD TO CASES IN WHICH A LINEARIZATION MAY BE ALLOWED

For the ship running in head seas or for the ship at zero speed the nonlinearity is not significant and therefore can be neglected. Now it will be demonstrated that the proposed method delivers useful results for this case.

For the model in regular waves, eq. 25 or eq. 26 must be used. But for cases in which the nonlinearity can be neglected, the more simple expression for $(I + iY)$ written in exp.19a can be used instead of the more complicated expression written in exp. 19b:

$$(I + iY) \approx \frac{\sin\left[\frac{\omega T}{2}\left(1 + \frac{\omega V}{g}\right)\right]}{\frac{\omega T}{2}\left(1 + \frac{\omega V}{g}\right)} \quad \text{for head seas} \tag{27}$$

Then eq. 26 for determining the maximum value of the average acceleration b or of the increment of velocity (bT) becomes:

$$bT + \frac{N}{m}\frac{b}{2}T^2 = \bar{b}T \; \frac{\sin\left[\frac{\omega T}{2}\left(1+\frac{\omega V}{g}\right)\right]}{\frac{\omega T}{2}\left(1+\frac{\omega V}{g}\right)} \tag{28}$$

The influence of the damping coefficient $\frac{N}{m}$ is small if application is restricted to the range below the first maximum. The maximum possible increment (for $\frac{N}{m} = 0$) amounts to:

$$(bT)_{max} = 2 \frac{\bar{b}}{\omega(1 + \frac{\omega V}{g})} \qquad (29a)$$

and will be reached in a time interval:

$$T = \frac{\pi}{\omega(1 + \frac{\omega V}{g})} \qquad \text{for } (bT)_{max} \qquad (29b)$$

Solution of the linearized equation of surging motion with zero damping force in a regular head sea gives:

1. an amplitude of oscillatory surging velocity one-half the value obtained from eq. 29a, and

2. a period double the value obtained from eq. 29b.

This confirms the relations for a harmonic oscillation, where the maximum increment of velocity equals two times the velocity amplitude, and it occurs during one-half of the period of oscillation, as shown in Fig. 3.

For the case in an irregular head sea, the spectrum on the right-hand side of eq. 23 becomes, without the non-linearity,

$$\omega^4 f^2 \left(\frac{\lambda}{L}\right) T^2 \left\{ \frac{\sin\left[\frac{\omega T}{2}(1+\frac{\omega V}{g})\right]}{\frac{\omega T}{2}(1 + \frac{\omega V}{g})} \right\}^2 r(\omega) \qquad (30)$$

This result, for a vanishing damping force, is in complete agreement with the result obtained in the normal way from the linearized equation of motion (eq. 7). Also, for the case of a vanishing damping force, exp. 30 represents the spectrum of the increment of velocity. On the other hand, eq. 8 is the result for the velocity of a straightforward solution of eq. 7. The increment in any time interval T, therefore, can be determined for this linearized case directly from eq. 8.

Increment of velocity = $\dot{x}_o (t + T) - \dot{x}_o (t)$ =

$$= \int_0^\infty \left\{ \cos\left[\omega(1+\frac{\omega V}{g})(t+T)+\sigma(\omega)\right] - \cos\left[\omega(1+\frac{\omega V}{g}) t + \sigma(\omega)\right]\right\} \sqrt{\frac{\omega^2 f^2(\frac{\lambda}{L}) r(\omega)}{(1+\frac{\omega V}{g})^2}} d\omega$$

$$= \int_0^\infty \cos\left[\omega(1+\frac{\omega V}{g}) t+\sigma_1(\omega)\right] \sqrt{\frac{\omega^2 f^2(\frac{\lambda}{L}) r(\omega)}{(1+\frac{\omega V}{g})^2} 4 \sin^2\left[\frac{\omega T}{2}(1+\frac{\omega V}{g})\right]} d\omega \tag{31}$$

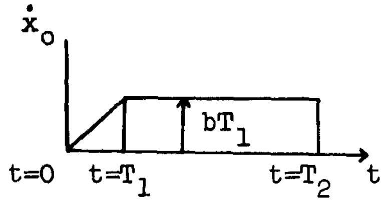
The spectrum of the increment of velocity found in this normal way is the same as the impulse spectrum (exp. 30).

For the cases discussed in this section and the previous one, there is no advantage in using the proposed method in place of the method developed in refs. 1 or 2. However, the former method can not be used in treating the surging motion in an irregular following sea since the influence of nonlinearity is not included.

IMPULSE FOR ANOTHER ASSUMPTION OF THE VELOCITY $\dot{x}_o$

In the section entitled "Impulse Spectrum," a motion with a constant acceleration b within the time interval T is assumed and then the impulse is calculated. It is known from refs. 2 and 3 that in a regular following sea the ship is accelerated only to the wave speed and after that, it runs with the wave for an unlimited time. Figure 2 also shows a similar trend.

Therefore, it is also interesting to calculate the impulse for a velocity $\dot{x}_o$ which remains constant after a certain increment of velocity is reached.



$$\dot{x}_o = b\tau \qquad \text{for } 0 \leq \tau \leq T_1$$
$$\dot{x}_o = bT_1 \qquad \text{for } T_1 \leq \tau \leq T_2 \tag{32}$$

For this case, exp. 18 must be supplemented by a second term:

$$\int_0^{T_1} e^{i\left[-\omega(1-\frac{\omega V}{g})\tau + \frac{\omega^2}{g}\frac{b}{2}\tau^2\right]} d\tau + \int_{T_1}^{T_2} e^{i\left[-\omega(1-\frac{\omega V}{g})\tau + \frac{\omega^2}{g}\frac{b}{2}T_1^2 + \frac{\omega^2}{g} b\, T_1(\tau - T_1)\right]} d\tau \tag{33}$$

Expression 33 then takes the place of exp. 19b to become the following:

$$\frac{e^{-i(1-\frac{\omega V}{g})^2 \frac{g}{2b}}}{\omega\sqrt{\frac{b}{2g}}} \left\{ \int_{-(1-\frac{\omega V}{g})\sqrt{\frac{g}{2b}}}^{\left[-(1-\frac{\omega V}{g})\sqrt{\frac{g}{2b}}+\omega T_1\sqrt{\frac{b}{2g}}\right]} e^{iu^2}\, du \right. \tag{34}$$

$$\left. + \frac{\sin\left[\frac{\omega}{2}(T_2-T_1)(-1+\frac{\omega V}{g}+\omega T_1\frac{b}{g})\right]}{\sqrt{\frac{g}{2b}}\left[-1+\frac{\omega V}{g}+\omega T_1\frac{b}{g}\right]} e^{i\left[-1+\frac{\omega V}{g}+\omega T_1\frac{b}{g}\right]\left[-1+\frac{\omega V}{g}+\omega T_2\frac{b}{g}\right]\frac{g}{2b}} \right\}$$

It may be supposed that the assumption of an interval of constant velocity following acceleration will lead to a prediction of a ship running at the dominant wave speed for an extended period of time. Instead of exp. 22, the resulting impulse spectrum becomes:

$$\omega^4\, f^2(\tfrac{\lambda}{L})\, T_1^{\,2}\, (I_2^{\,2} + Y_2^{\,2})\, r(\omega) \tag{35}$$

where $T_1{}^2 \left[I_2{}^2 + Y_2{}^2 \right]$ represents the square of the absolute value of exp. 34.

Discussion of the irregular sea case will be continued later; the following applies only to a regular sea. It is assumed that the ship is accelerated to the wave speed and the question is whether or not the ship will continue to run at the wave speed. (wave speed $= \frac{g}{\omega}$)

$$bT_1 = \frac{g}{\omega} - V \tag{36}$$

This equation, put in exp. 34, results in:

$$\frac{e^{-i\left(1-\frac{\omega V}{g}\right)^2 \frac{g}{2b}}}{\omega\sqrt{\frac{b}{2g}}} \left\{ \int_{-\left(1-\frac{\omega V}{g}\right)\sqrt{\frac{g}{2b}}}^{0} e^{iu^2}\, du + \omega\sqrt{\frac{b}{2g}}\,(T_2 - T_1) \right\} \tag{37}$$

This means that during the second time interval $T_1 < \tau < T_2$, the impulse grows linearly with the time, or in other words, the force in the longitudinal direction remains constant. Therefore, if the condition of eq. 26 can be fulfilled for the first time interval, the ship can run with the wave an infinite time. Also this result is very close to the results obtained in refs. 2 and 3.

A mean wave speed will be dominant in an irregular sea. If the model can be accelerated by the sea to this mean wave speed, a long run at this wave speed appears possible. Therefore, it can be expected that the approach developed in this section will be useful for the case in an irregular sea.

SOME RESULTS FOR NONLINEAR BEHAVIOR IN AN IRREGULAR FOLLOWING SEA

In the fourth and fifth sections it was shown that the proposed method leads to useful results for the oscillatory surging motion in an irregular head sea as well as for the striking nonlinear behavior of a model in regular following waves. Now the method will be used to investigate the nonlinear behavior in an irregular following sea.

The numerical computations are carried out on the IBM 1620 electronic computer at Stevens Institute of Technology.

A two-dimensional modified Neumann spectrum is assumed for the irregular sea:

$$r(\omega) = \frac{\pi}{2}\frac{C}{\omega^6} e^{-\frac{2g^2}{u^2\omega^2}} \tag{38}$$

The constant C amounts to 32 $ft^2\ sec^{-5}$ for the Neumann spectrum, but for the computations performed for this study, C was reduced by the factor 0.75 to 24 $ft^2\ sec^{-5}$. The influence of the three-dimensional nature of the real sea on the surging force is estimated by this reduction.

Equation 3 is used for the surging force function. Then the impulse spectrum amounts to (see exp. 22):

$$\frac{\pi}{2}\frac{C}{\omega^2} e^{-\frac{2g^2}{u^2\omega^2}} \left[3\,\frac{\sin\left(\frac{\omega^2L}{2g}\right)-\left(\frac{\omega^2L}{2g}\right)\cos\left(\frac{\omega^2L}{2g}\right)}{\left(\frac{\omega^2L}{2g}\right)^3}\right]^2 T^2(I^2+Y^2) \tag{39}$$

where $T^2(I^2 + Y^2)$ represents the square of the absolute value of the integral (exp. 18). It can be taken as in:

1. exp. 19a, if nonlinearity is excluded,

2. exp. 19b, if nonlinearity is included and eq. 16 is used for the assumed motion,

3. exp. 34, if the nonlinearity is included and eq. 32 is used for the assumed motion.

The area under the impulse spectrum is determined by:

$$\frac{9}{2}\pi CT^2 \int_0^\infty \frac{e^{-\frac{2g^2}{u^2\omega^2}}}{\omega^2} \left[\frac{\sin\left(\frac{\omega^2 L}{2g}\right) - \left(\frac{\omega^2 L}{2g}\right)\cos\left(\frac{\omega^2 L}{2g}\right)}{\left(\frac{\omega^2 L}{2g}\right)^3} \right]^2 (I^2+Y^2)\, d\omega \tag{40}$$

This area depends on the wind speed u, ship speed V, ship length L and the assumed motion. The root of this area is written as:

$$\sqrt{\frac{9}{2}\pi\, CT^2 \int_0^\infty \ldots d\omega} = C^{1/2}\left(\frac{L}{g}\right)^{3/4} D \tag{41}$$

where C = the reduced factor for the Neumann-spectrum = $24\ ft^2\ sec^{-5}$

L = the length of the ship in feet

g = the gravitational acceleration

D = a dimensionless number

D is computed from the impulse spectrum as a function of the assumed motion, of the Froude number $V/\sqrt{gL}$ for the speed of the ship and of the dimensionless parameter $u/\sqrt{gL}$ for the wind speed.

For the assumed motion, the following must be chosen:

In Case 1. only the time interval T (see exp.19a). (This case is computed only as a basis for comparison.)

In Case 2. the time interval T and the acceleration b (see exp.19b).

In Case 3. the time interval T_1, the acceleration b and the time interval T_2 (see exp. 34).

The impulse is a random value. If a Rayleigh distribution is assumed, the amplitude of the impulse therefore amounts to:

$$aC^{1/2}\left(\frac{L}{g}\right)^{3/4}D \tag{42}$$

where a = 0.886 for the mean value of the amplitudes
a = 1.41 for the average of the 1/3 highest amplitudes
a = 1.80 for the average of the 1/10 highest amplitudes
a = 2.25 for the average of the 1/100 highest amplitudes

In each case, the impulse must equal the left-hand side of eq. 23. The damping term in this equation may be neglected for many cases. In such cases, it follows from eq. 23 and exp. 42 that:

$$bT = aC^{1/2}\left(\frac{L}{g}\right)^{3/4}D \tag{43}$$

or

$$\frac{\sqrt{gL}}{bT}D = \frac{g^{5/4}}{aC^{1/2}L^{1/4}} \tag{44}$$

If the nonlinearity is excluded as in Case 1, the value D is independent of the acceleration b. Then this value D can be computed as a function of the Froude number, the dimensionless wind speed and the time interval T. Figure 4 shows the results of such a computation. The increment of velocity (bT) can be determined from this figure for any ship's length L and for any value, "a." If the nonlinearity is included as in Case 2, the left-hand side of eq. 44 must be determined first. Only the left-hand side of this equation depends on the assumed motion. Fig. 5 shows the result of such a computation for Case 2. Fig.5 is valid only for $Fr=0.2/\sqrt{2}$ and $u^2/gL=1/3$. But the assumed motion is

varied by varying the parameters $\sqrt{\frac{g}{2b}}$ and T $\sqrt{b/L}$. Now it can be determined from this figure which assumed motion is possible for a given right-hand side of eq. 44. For instance, if a ship of L = 220 ft is given and 2.25 is chosen as factor "a," the value of the right-hand side of eq. 44 amounts to:

$$\frac{32^{5/4}}{2.25 \times 24^{1/2} \times 220^{1/4}} = 1.79$$

All points in Fig. 5 having an ordinate of 1.79 represent for the given ship, Froude number and wind speed, an average of the 1/100-highest motions. If 1.80 is chosen as the value of the factor "a," the right-hand side of eq. 44 amounts to 2.24 and all points in Fig. 5 having such an ordinate represent an average of the 1/10-highest motions. Figure 6 shows both of these motions and it also shows the same motions determined without the influence of the nonlinearity and computed from Fig. 4. The same procedure is carried out for other wind speeds and Froude numbers. Figure 7 shows results analogous to Fig. 6 but for a more severe sea.

Figures 6 and 7 illustrate that the influence of nonlinearity is stronger, the more severe the sea is or the larger the factor "a" is. The average of the 1/100-highest motions will be a larger motion than the average of the 1/10-highest motions and the influence of the nonlinearity must be stronger for the larger motion. Therefore, the ratio of the average of the 1/100-highest motions to the average of the 1/10-highest motions is larger than the ratio of the "a" values (2.25/1.80 = 1.25) for the nonlinear motion. Figure 8 shows results for other wind speeds.

The damping term on the left-hand side of eq. 23 is neglected in computations for Figs. 4 through 8. The motion will be reduced by this term. It is supposed that this

influence is unimportant for $u^2/gL = 1/5$ and $1/4$ in Fig. 8, but may be important for $u^2/gL = 1/3$ and $1/2$ in this figure. However, this influence will be discussed later. Now it may be concluded only that for a Froude number of $0.2/\sqrt{2}$, the increment of the velocity becomes very large for wind speeds of $u^2/gL = 1/3$ or more.

In the next step Case 3 is treated, that is, eq. 32 is taken for the assumed motion. First the damping term on the left-hand side of eq. 23 is neglected. Then eqs. 43 and 44 must be fulfilled for this case. The difference compared to Case 2 is only that T in these equations must be replaced by T_1 and that the assumed motion is expressed in D by the parameters b, T_1 and T_2. It appears best to use the results already obtained for Case 2.

In Case 2 a probable motion $x_o = b\tau^2/2$ or $\dot{x}_o = b\tau$ was obtained in the time interval T for given values of the Froude number Fr, the wind speed u, the length of the ship L, and the value "a."

It is now asked what the impulse is, if it is assumed that for the same pre-conditions, after the time interval $\tau = T_1$, the ship runs with a constant velocity $(V + bT_1)$. The following form of eqs. 43 or 44 is used to represent the results of such a computation:

$$\frac{bT}{\sqrt{gL}} = aC^{1/2} \frac{L^{1/4}}{g^{5/4}} D \qquad (45)$$

The right-hand side of this equation is computed from the impulse spectrum and it is plotted versus the time $T_2 \sqrt{g/L}$ in Fig. 9. The left-hand side of this equation is determined from the assumed motion and plotted as dashed

lines in this figure. Equation 45 is fulfilled only if the corresponding curves in the figure agree.

This agreement may be close enough for the first time interval. The results for this time interval do not agree exactly with the corresponding results represented in Figs. 6 to 8. The reason for this discrepancy is only that to save time the computations are not repeated with the parameters b and T which can be read from Fig. 8.

However, the agreement of corresponding curves in Fig. 9 is not close enough for the time $\tau > T_1$.

In spite of these discrepancies, it can be concluded that for the Froude number $F_r = 0.2/\sqrt{2}$, the ship's length L = 220 ft and for wind speeds of $u^2/gL = 1/5$ and $1/4$, the ship will run with a higher speed only a short time. On the other hand, it is possible that for $u^2/gL = 1/3$ and $1/2$, the real motion lies somewhere between the computed and the assumed motion, and also that the ship will run with a higher speed a long time.

It may be mentioned at this point that the curves for the computed motion and for $u^2/gL = 1/3$ and $1/2$ in Fig. 9 for a time τ steadily increasing will reach a maximum and after that will descend. However, further discussion should include the influence of the damping term in eq. 23.

Furthermore, it may be mentioned that the assumption for Case 3 seems to lead to very significant results for behavior in an irregular sea.

INFLUENCE OF SHIP'S RESISTANCE ON NONLINEAR BEHAVIOR

The damping term on the left-hand side of eq. 23 is due to the resistance of the ship. This term is neglected in the preceding numerical computation. But such an omission is not allowable for cases in which the ship runs at a high speed for a long time.

Moreover, the increment of velocity may be very large (Fig. 9) and therefore the linearized form of the damping term used in eq. 23 may not be correct for such large increments of velocity.

For estimating the influence of damping, a quadratic law is assumed for the resistance. The damping term on the left-hand side of eq. 23 represents the difference between resistance R and propeller thrust T.

For the following computation it is assumed

$$R-T = \frac{R_V}{V^2} \left[(V + \dot{x}_o)^2 - V^2 \right] \qquad (46)$$

where R_V = a constant value. This value will not equal the resistance of the ship for the speed V because the ship's propeller will run with about a constant number of revolutions even when the ship is accelerated by the waves. The value R_V in eq. 46 will therefore be larger than the resistance for the speed V, and it must be determined by a resistance test of a model equipped with a propeller running with about a constant number of revolutions.

The left-hand side of eq. 13 is replaced by:

$$\ddot{x}_o + \frac{R_V}{mV^2} \left[(V+\dot{x}_o)^2 - V^2 \right] = \ddot{x}_o + \frac{R_V}{mV^2} (2V\dot{x}_o + \dot{x}_o^2) \qquad (47)$$

For Case 3, which has already been treated, a motion determined by eq. 32 is assumed:

$$\dot{x}_o = b\tau \qquad \text{for} \qquad 0 \leq \tau \leq T_1$$

$$\dot{x}_o = bT_1 \qquad \text{for} \qquad T_1 \leq \tau \leq T_2$$

The integration of eq. 47, similar to the integration of the left-hand side of eq. 14, leads to:

$$\int_0^{T_2} \left[\ddot{x}_o + \frac{R_v}{mV^2}(2V\dot{x}_o + \dot{x}_o^2)\right] d\tau = bT_1 \left\{ 1 + \frac{R_v}{mV^2}\left[2V(T_2 - \frac{T_1}{2}) + bT_1(T_2 - \frac{2}{3}T_1)\right]\right\} \tag{48}$$

The left-hand side of eq. 23 will therefore be replaced by eq. 48. The frequency ω does not appear in this expression and therefore it is only necessary in eq. 43 to 45 to divide D by

$$1 + \frac{R_v}{mV^2}\left[2V\left(T_2 - \frac{T_1}{2}\right) + bT_1\left(T_2 - \frac{2}{3}T_1\right)\right] \tag{49}$$

or, represented by nondimensional values:

$$1 + \frac{R_v}{mgF_r^{\,2}}\left[2F_r\left(T_2\sqrt{\frac{g}{L}} - \frac{T_1}{2}\sqrt{\frac{g}{L}}\right) + \frac{bT_1}{\sqrt{gL}}\left(T_2\sqrt{\frac{g}{L}} - \frac{2}{3}T_1\sqrt{\frac{g}{L}}\right)\right] \tag{50}$$

The new parameter $\frac{R_v}{mgF_r^{\,2}}$ is introduced. Unfortunately the magnitude of this parameter will vary for different ships. Therefore, the following numerical results will show only a general trend, and these results should not be used for any actual ship. $R_v/mgF_r^{\,2} = 0.15$ is used for the following numerical computations. This may be a large value, but

it must be remembered that the influence of the propeller is included in this value.

Figure 10 is obtained by dividing the computed values plotted in Fig.9 by the factor from exp. 50. A comparison of the two figures shows that the influence of the damping term is relatively small during the first time interval but that it is of great importance for the next time interval. Again it can be seen from Fig. 10 that in a mild sea, a high speed is possible only for a short time. It should be considered accidental that for $u^2/gL = 1/3$ the computed motion is so close to the assumed motion. If the increment of velocity for $u^2/gL = 1/3$ and $1/2$ in Fig. 10 appears unrealistically high, it should be remembered that analogous results were presented in refs. 2 and 3 for the case in regular waves. Also in ref. 2, a very high increment of velocity was found for a severe sea and even in ref. 3 a relatively high increment was found, even though a mild sea was used in the experiments. For instance, in Fig. 14 of ref. 3 an increment of $bT/\sqrt{gL}$ =0.166 is found for a wave speed of 8.9 ft/sec although the ratio of wave height to wave length was only 1:41.

AN ESTIMATE OF THE MAXIMUM DURATION OF A RUN WITH A HIGHER SPEED THAN THE CALM WATER SPEED

Figure 10 indicates that when $u^2/gL = 1/3$ and $1/2$, it is possible for the ship to maintain a high speed for a long interval. For cases in which the second time interval $(T_2 - T_1)$ is very long, it appears possible to make an estimate of this time interval by a simple method.

Using the case of $u^2/gL = 1/2$ in Fig. 10 as an example, the problem is to determine the time at which the curves of the computed motion and the assumed motion will intersect. The method used in this section is not expected

to result in a very accurate estimate, but a rough approximation only. Nevertheless, the results should help broaden our understanding of the nonlinear behavior of a ship.

The method of estimation is based on the fact that the factor $T_1^2\ (I_2^2 + Y_2^2)$ in the impulse spectrum for a long time $(T_2 - T_1)$ can be approximated by (see exps. 34 and 35):

$$T_1^2\ (I_2^2 + Y_2^2) \approx \left[\frac{\sin\left[\frac{\omega}{2}\ (T_2 - T_1)\ \left(-1 + \frac{\omega V}{g} + \omega T_1\ \frac{b}{g}\right)\right]}{\frac{\omega}{2}\ \left(-1 + \frac{\omega V}{g} + \omega T_1\ \frac{b}{g}\right)}\right]^2 \tag{51}$$

This is because for

$$\left(-1 + \frac{\omega V}{g} + \omega T_1\ \frac{b}{g}\right) \to 0 \tag{52}$$

the part of $T_1^2\ (I_2^2 + Y_2^2)$ represented by eq. 51 becomes a very large value. Although this part of $T_1^2\ (I_2^2 + Y_2^2)$ decreases quickly, if ω becomes smaller or larger as is needed for exp. 52, this part will be the dominant part in the cases mentioned. The integrand in exp. 40 will be relatively small for all values of ω except a very small range near

$$\omega_o = \frac{1}{\frac{V}{g} + T_1\ \frac{b}{g}} = \frac{g}{V + bT_1} \tag{53}$$

For such a condition it will be allowable to replace exp. 40 by:

$$\frac{9}{2}\pi C\ T_1^2\ \frac{e^{-\frac{2g^2}{u^2\omega_o^2}}}{\omega_o^2} \left[\frac{\sin\left(\frac{\omega_o^2 L}{2g}\right) - \left(\frac{\omega_o^2 L}{2g}\right)\cos\left(\frac{\omega_o^2 L}{2g}\right)}{(\omega_o^2 L/2g)^3}\right]^2$$
$$\int_{-\infty}^{+\infty} \left[\frac{\sin\left[\frac{\omega_o}{2}\ (T_2 - T_1)\ \left(-1 + \frac{\omega V}{g} + \omega T_1\ \frac{b}{g}\right)\right]}{\frac{\omega_o T_1}{2}\ \left(-1 + \frac{\omega V}{g} + \omega T_1\ \frac{b}{g}\right)}\right]^2 d\omega \tag{54}$$

The integration is carried out and the following is obtained:

$$9\pi^2\, C\, (T_2-T_1)\, \frac{e^{\frac{-2g^2}{u^2\omega_o^2}}}{\omega_o^{\,2}} \left[\frac{\sin\left(\frac{\omega_o^2 L}{2g}\right) - \frac{\omega_o^2 L}{2g}\cos\left(\frac{\omega_o^2 L}{2g}\right)}{\left(\frac{\omega_o^2 L}{2g}\right)^3}\right]^2 \tag{55}$$

The factor from exp.50 for the left-hand side of eq. 23 is approximated for a very large value of T_2 by

$$\frac{R_v}{mgF_r^{\,2}} \sqrt{\frac{g}{L}}\,(T_2 - T_1) \left(2F_r + \frac{bT_1}{\sqrt{gL}}\right) \tag{56}$$

Now, both of these approximations are used to establish the following equation in place of eq. 43:

$$bT_1 \frac{R_v}{mgF_r^{\,2}} \sqrt{\frac{g}{L}}\,(T_2-T_1) \left(2\, F_r + \frac{bT_1}{\sqrt{gL}}\right) = a\sqrt{9\pi^2\, C(T_2-T_1)\frac{e^{\frac{-2g^2}{u^2\omega_o^2}}}{\omega_o^2}\left[\frac{\sin\left(\frac{\omega_o^{\,2}L}{2g}\right)-\left(\frac{\omega_o^{\,2}L}{2g}\right)\cos\left(\frac{\omega_o^2 L}{2g}\right)}{\left(\frac{\omega_o^{\,2}L}{2g}\right)^3}\right]^2} \tag{57}$$

The time interval $(T_2 - T_1)$ follows from that equation:

$$\sqrt{\frac{g}{L}}\quad (T_2-T_1) \approx 18\pi^2\, a^2\, \frac{C\, L^{1/2}}{g^{5/2}} \left(\frac{mg\, F_r^{\,2}}{R_v}\right)^2 \frac{e^{-\frac{gL}{u^2}\frac{2g}{\omega_o^2 L}}}{\left(\frac{\omega_o^{\,2}L}{2g}\right)^5} \left[\frac{\sin\left(\frac{\omega_o^2 L}{2g}\right) - \left(\frac{\omega_o^2 L}{2g}\right)\cos\left(\frac{\omega_o^2 L}{2g}\right)}{1 - 2\,\frac{\omega_o^2 L}{2g}\, F_r^{\,2}}\right]^2 \tag{58}$$

These results are plotted in Fig. 11 versus $(\omega_0^2 L/2g)$. In the upper part of this figure, the increment of velocity computed from eq. 53 is plotted and in the lower part the time computed from eq. 58 is plotted.

The following three steps must be made in order to understand Fig. 11:

1. It must be determined to which increment of velocity the ship can be accelerated by the sea. Such a determination will lead to results like those plotted in Figs. 5 to 8.

2. The value $\omega_0^2 L/2g$ must be determined from the upper part of Fig. 11 for the increment of velocity just found.

3. An estimate of the time for the run of the ship at increased speed can be obtained for this value of $\omega_0^2 L/2g$ from the lower part of Fig. 11.

For instance, for the case $u^2/gL = 1/2$ represented in Fig. 10, the increment of velocity produced in the first interval amounts to 0.35. For this increment the value 2.04 for $\omega_0^2 L/2g$ is obtained from the upper part of Fig. 11 and for the same abscissa, the time $(T_2 - T_1)\sqrt{g/L} = 73.5$ is determined. Therefore, both curves plotted in Fig. 10 for $u^2/gL = 1/2$ will intersect after the very long time of 73.5 and the real motion may lie anywhere between the two plotted motions.

The same procedure delivers a time $(T_2 - T_1)\sqrt{g/L}=40$ for the case $u^2/gL = 1/3$ represented in Fig. 10.

For the other cases, $u^2/gL = 1/4$ and $1/5$, this estimate is not possible because the increment of velocity produced in the first time interval is too small and lies outside of Fig. 11.

Figure 11 may also be used to find the increment of velocity which will result in the longest run at high speed. The probability of such a long run occurring is somewhat

smaller than that implied by the factor "a." This is because the equation used in this computation is subject only to the condition that the assumed motion and the computed motion coincide at the end points of the time interval. The probability could be maintained only if the assumed and computed motions were the same over the entire time interval. On the other hand, it might be possible to estimate the decrease in the probability of obtaining a long run, but it is believed that the results of such an estimate would not be of great importance.

CONCLUSIONS

1. It appears impossible to determine the surging motion in a severe following sea without including the nonlinearity in such a determination.

2. An enormous change in the trend of the surging motion due to the nonlinearity seems possible.

3. A large increase in speed and a run at this higher speed over a relatively long interval of time can be produced by a following sea on a ship running slower than the dominant waves.

4. The increment of speed and the duration of the run at higher speed can be estimated by the method described here.

5. The surging motion itself is not dangerous for the ship. But it is believed that the nonlinear surging motion treated here may be connected with dangerous motions in other modes. It is believed that the treatment of surging motion presented here will provide a valuable starting point for future investigations of other more dangerous motions.

BROACHING OF A SHIP IN A SEVERE IRREGULAR FOLLOWING OR QUARTERING SEA

INTRODUCTION

Broaching of ships in severe following or quartering seas has been observed and examples of broaching are described in ref. 3. Broaching of a ship is very dangerous and must be avoided. Therefore it is important to know the conditions which lead to such an occurrence.

The problem was discussed in connection with nonlinear surging motion in refs. 2 and 3. However, broaching is a very complicated problem and not even an approximate solution is known.

Some aspects of the problem are discussed in this part of the report. Neither a complete solution nor a complete representation can be presented. However, discussion of the problem appears justified because results for nonlinear surging motion in an irregular sea are now available.

The problem is discussed for only a long-crested sea and also for only small heading angles. It is hoped that the reasons for broaching can be recognized by an investigation within these specified limits.

THE EXCITING FORCES FOR THE SURGING, SWAYING AND YAWING MOTIONS

It is assumed that the important nonlinearities are not connected with the determination of the exciting forces but with the solutions of the equations of motion. In particular, it is assumed that the important nonlinearities are connected with the displacement of the ship relative to the wave pattern. This was demonstrated in part I for the surging motion in a following sea.

No new method is used in determining the exciting forces. Certainly, there are difficult hydrodynamic problems connected with an accurate determination of these forces. However, the aim of this report is not to treat these hydrodynamic problems but to treat the nonlinear equations of motion. Therefore, only the Froude-Kryloff hypothesis is used for determining the exciting forces. It should be mentioned that the computation of the surging force in this way is supposed to be a good approximation, while it is thought that the yawing moment and the swaying force computed in this way will be too small. This supposition is supported by the fact that, for a two-dimensional body in a beam wave, the computed swaying force is too small if the Froude-Kryloff hypothesis is used for such a computation. If the swaying force or yawing moment is computed by this simple method, it is estimated that the result should be multiplied by a factor of 2.

Although the exciting forces resulting from the Froude-Kryloff hypothesis are used for the numerical results, the foregoing remarks about such results should be borne in mind.

Analogous to eq. 3, the following equations are used for the exciting forces in a regular quartering sea (see Fig. 12):

$$\text{surging force} = \frac{2\bar{h}}{\lambda}\pi \cos\psi \, \nabla \, f_{xx}\left(\frac{\lambda}{L\cos\psi}\right)\cos\left[\frac{\omega^2 x_o}{g\cos\psi} - \omega\left(1-\frac{\omega V}{g\cos\psi}\right)t+\sigma\right]$$

$$\text{swaying force} = \frac{2\bar{h}}{\lambda}\pi \sin\psi \, \nabla \, f_{yy}\left(\frac{\lambda}{L\cos\psi}\right)\cos\left[\frac{\omega^2 x_o}{g\cos\psi} - \omega\left(1-\frac{\omega V}{g\cos\psi}\right)t+\sigma\right]$$

$$\text{yawing moment} = \frac{2\bar{h}}{\lambda}\pi \sin\psi \, \nabla L f_{zz}\left(\frac{\lambda}{L\cos\psi}\right)\sin\left[\frac{\omega^2 x_o}{g\cos\psi} - \omega\left(1-\frac{\omega V}{g\cos\psi}\right)t+\sigma\right]$$

(59)

It will be permissible to replace $\cos\psi$ by 1.0 and $\sin\psi$ by ψ for a small range of the heading angle ψ.

The force functions of $(\frac{\lambda}{L})$ are real functions for a body symmetrical about the midship section. Furthermore, the functions $f_{xx}(\frac{\lambda}{L})$ and $f_{yy}(\frac{\lambda}{L})$ will be identical. For a parabolic sectional area curve, the following expressions are valid:

$$f_{xx}\left(\frac{\lambda}{L}\right) = f_{yy}\left(\frac{\lambda}{L}\right) = 3\frac{\sin\left(\frac{\omega^2 L}{2g}\right) - \left(\frac{\omega^2 L}{2g}\right)\cos\left(\frac{\omega^2 L}{2g}\right)}{\left(\frac{\omega^2 L}{2g}\right)^3}$$

$$f_{zz}\left(\frac{\lambda}{L}\right) = \frac{3}{2}\cdot\frac{\left[3-\left(\frac{\omega^2 L}{2g}\right)^2\right]\sin\left(\frac{\omega^2 L}{2g}\right) - 3\left(\frac{\omega^2 L}{2g}\right)\cos\left(\frac{\omega^2 L}{2g}\right)}{\left(\frac{\omega^2 L}{2g}\right)^4} \qquad (60)$$

These expressions for the exciting forces are used for the following numerical computations.

For the case in a long-crested irregular sea, eqs. 59 are replaced by the following:

$$F_x = \text{surging force} = \nabla\cos\psi\int_0^\infty \cos\left[\frac{\omega^2 x_o}{g} - \omega\left(1-\frac{\omega V}{g}\right)t + \sigma(\omega)\right]\sqrt{r\frac{\omega^4}{g^2}f_{xx}^2\left(\frac{\lambda}{L}\right)d\omega}$$

$$F_y = \text{swaying force} = \nabla\sin\psi\int_0^\infty \cos\left[\frac{\omega^2 x_o}{g} - \omega\left(1-\frac{\omega V}{g}\right)t + \sigma(\omega)\right]\sqrt{r\frac{\omega^4}{g^2}f_{xx}^2\left(\frac{\lambda}{L}\right)d\omega}$$

(eq. 61 con't on next pg)

$$F_z = \text{yawing moment} = \nabla L \sin\psi \int_0^\infty \sin\left[\frac{\omega^2 x_o}{g} - \omega\left(1 - \frac{\omega V}{g}\right) t + \sigma(\omega)\right]$$

$$\sqrt{r \frac{\omega^4}{g^2} f_{zz}^2 \left(\frac{\lambda}{L}\right) d\omega} \qquad (61)$$

It is important to note that the following ratio depends only on the heading angle in an irregular long-crested sea as well as in regular waves.

$$\frac{\text{swaying force}}{\text{surging force}} = \tan\psi$$

Nondimensional spectra are computed assuming a Neumann spectrum for the long-crested irregular sea and plotted versus $\omega\sqrt{\frac{L}{g}}$ in Fig. 13.

EQUATIONS OF THE SWAYING AND YAWING MOTIONS

For investigations of coursekeeping or steering in calm water, the following linearized equations are commonly used:

$$\begin{aligned} -m_y\dot{\beta} + (m_x - Y_r)\dot{\psi} - Y_\beta\beta &= Y_\delta\delta \\ n_z\ddot{\psi} - N_\beta\beta - N_r\dot{\psi} &= N_\delta\delta \end{aligned} \qquad (62)$$

As a criterion of course stability in calm water, the following condition is known:

$$Y_\beta N_r + (m_x - Y_r) N_\beta < 0 \qquad (63)$$

Now it must be decided what supplements are needed for a realistic investigation of the behavior in a severe irregular quartering sea. First the following nondimensional expressions for the exciting swaying force and exciting yawing moment must be added to eq. 62:

to the first equation:

$$\frac{F_y}{\frac{\rho}{2}LHV^2} = \frac{m_x}{F_r^2}\sin\psi\int_0^\infty \cos\left[\frac{\omega^2 x_o}{g} - \omega\left(1-\frac{\omega V}{g}\right)t + \sigma(\omega)\right]\sqrt{r\,\frac{\omega^4}{g^2}\,f_{xx}^2\left(\frac{\lambda}{L}\right)\,d\omega}$$

to the second equation: (64)

$$\frac{F_z}{\frac{\rho}{2}L^2HV^2} = \frac{m_x}{F_r^2}\sin\psi\int_0^\infty \sin\left[\frac{\omega^2 x_o}{g} - \omega\left(1-\frac{\omega V}{g}\right)t + \sigma(\omega)\right]\sqrt{r\,\frac{\omega^4}{g^2}\,f_{zz}^2\left(\frac{\lambda}{L}\right)\,d\omega}$$

Another supplement appears necessary. In finding eqs. 62 in ref. 5, a constant speed of the ship is assumed. However, a ship in waves always will be accelerated or decelerated in the longitudinal direction. The influence of such an acceleration is included in eqs. 4 in ref. 5 and it is evident that this influence will be important for the discussions in this report. The nondimensional expression $\frac{V'}{V}$ is used by Davidson in ref. 5 for the acceleration in a longitudinal direction. In part I of this report, the acceleration b equals the right-hand side of eq. 23 if the damping term is neglected. Using this result, the non-dimensional acceleration amounts to:

$$\frac{V'}{V} = \frac{bL}{V^2} = \frac{1}{F_r^2}\int_0^\infty \cos\left[\frac{\omega^2}{g}x_o - \omega\left(1-\frac{\omega V}{g}\right)t + \sigma(\omega)\right]\sqrt{r\frac{\omega^4}{g^2}f_{xx}^2\left(\frac{\lambda}{L}\right)\,d\omega} \tag{65}$$

The supplemented equations now can be written as:

$$-m_y\frac{V'}{V}\beta - m_y\dot{\beta} + (m_x - Y_r)\dot{\psi} - Y_\beta\beta = Y_\delta\delta - m_x\psi\frac{V'}{V}$$

$$n_z\frac{V'}{V}\dot{\psi} + n_z\ddot{\psi} - N_\beta\beta - N_r\dot{\psi} =$$

$$N_\delta\delta + \frac{m_x}{F_r^2}\psi\int_0^\infty \sin\left[\frac{\omega^2 x_o}{g} - \omega\left(1-\frac{\omega V}{g}\right)t + \sigma(\omega)\right]\sqrt{r\frac{\omega^4}{g^2}f_{zz}^2\left(\frac{\lambda}{L}\right)d\omega} \tag{66}$$

It is believed that these supplements are sufficient for a discussion of broaching.

The equations are pseudolinear because the coefficients of two terms in each equation are non-constant, being dependent on time. It is not possible to linearize these equations and still achieve the objectives of this report. A complete solution of these equations is not known and it is not the intent of this report to develop such a solution.

AN ESTIMATE OF COURSE KEEPING IN A SEVERE FOLLOWING OR QUARTERING SEA

An attempt could be made to replace the non-constant coefficients in the equations by harmonic functions. However, a study of Fig. 13 shows that such an approximation would lead to a frequency for the exciting yawing moment different than that for the surging or swaying exciting force. Therefore a solution of the equation would be very complicated for such an assumption.

Another simplification of the problem will be used for a rough estimate. To estimate the influence of the yawing exciting moment, a ship running with a constant speed determined by a Froude number of about 0.44 is considered. Figure 13 shows that for such a run the average period of the yawing exciting moment will be the largest possible period. The influence of the yawing moment is supposed to be the largest for this case (because the frequency of encounter then equals zero for $\omega\sqrt{L/g} = 2.3$). Then the coefficient of the yawing exciting moment in eq. 66 is replaced by a constant coefficient:

$$\frac{m_x}{{F_r}^2}\psi a\sqrt{\int_0^\infty r\frac{\omega^4}{g^2}f_{zz}^2\left(\frac{\lambda}{L}\right)d\omega} \qquad (67)$$

During the same time interval the surging and swaying exciting forces are unknown. Perhaps the probability for different forces connected with the moment of exp. 67 could be found by a complicated investigation. This will not be attempted, however, and therefore the forces are neglected for the case considered here. In reality, the course keeping will be improved in some cases and deteriorated in other cases by these forces. Therefore, in some cases the course keeping will be worse than estimated when neglecting surging or swaying exciting forces.

Steering is necessary to avoid an unstable run of the ship. The simplest assumption is chosen for steering. That is, the rudder angle δ is proportional to the difference between the heading angle ψ to the waves and the desired heading angle ψ_o:

$$\delta = \gamma(\psi - \psi_o) \tag{68}$$

The minimum factor γ for steering, to avoid an unstable run of the ship, will be computed and used as a yardstick for the possibilities of steering in the cases discussed.

Now, the first very simplified case follows:

$$-m_y \dot{\beta} + (m_x - Y_r)\dot{\psi} - Y_\beta \beta = Y_\delta \gamma(\psi - \psi_o)$$

$$n_z \ddot{\psi} - N_\beta \beta - N_r \dot{\psi} = N_\delta \gamma(\psi - \psi_o) \pm \frac{m_x}{F_r^2} \psi a \sqrt{r \frac{\omega^4}{g^2} f_{zz}^2 \left(\frac{\lambda}{L}\right) d\omega} \tag{69}$$

"a" is chosen as 2.25 for the average of the 1/100 most severe cases. The spectrum is taken as in Fig. 13. The ship's parameters are taken from ref. 6 and ships A and C mentioned in this reference are chosen. Then the equations deliver the following minimum factor γ:

			for ship A	for ship C
for $\frac{u^2}{gL} = \frac{1}{2}$;	F_r=0.45;	γ min. =	0.51	0.82
for $\frac{u^2}{gL} = \frac{1}{3}$;	F_r=0.44;	γ min. =	0.42	0.67
for $\frac{u^2}{gL} = \frac{1}{4}$;	F_r=0.43;	γ min. =	0.36	0.58

Another case is discussed before any conclusions are drawn from these results. A constant acceleration of the ship, and therefore also a constant swaying exciting force is possible, over a relatively long time interval, even for a low ship speed. That is shown in Fig. 10 for a Froude number of 0.142. Such an accelerated run of a ship is considered as the next case. The yawing exciting moment is unknown during such an accelerated run. This moment is therefore neglected. The same remarks mentioned in the preceding case for neglecting the forces could now be made for neglecting the moments. For this second very simplified case, the following equations are used:

$$-m_y\frac{V'}{V}\beta - m_y\beta' + (m_x - Y_r)\dot{\psi} - Y_\beta\beta = Y_\delta\gamma(\psi-\psi_o) - m_x\psi\frac{V'}{V} \qquad (70)$$

$$n_z\frac{V'}{V}\dot{\psi} + n_z\ddot{\psi} - N_\beta\beta - N_r\dot{\psi} = N_\delta\gamma(\psi-\psi_o)$$

where
$$\frac{V'}{V} = \frac{a}{F_r^{\,2}}\sqrt{\int_0^\infty r\frac{\omega^4}{g^2}f_{xx}^2\left(\frac{\lambda}{L}\right)d\omega} \qquad (71)$$

The computations are carried out for the same ship's parameter as in the preceding case. The following results for the necessary factor γ are obtained:

				for ship A	for ship C
for $\frac{u^2}{gL} = \frac{1}{2}$;	F_r=0.142;	$\frac{V'}{V}$ = 8.7;	γ min.	1.0	2.4
for $\frac{u^2}{gL} = \frac{1}{3}$;	F_r=0.142;	$\frac{V'}{V}$ = 5.8;	γ min.	0.9	2.1

The minimum factor γ is much larger for this case than for the preceding one. Therefore it is concluded that the latter case is more important if broaching is considered.

It could be demonstrated that an accelerated run lasts even longer for a smaller Froude number than for a larger one. However, there will be a limit. If the Froude number decreases under this limit, only a short accelerated run will be possible. The problem of course keeping will be the most severe for a ship running with the afore-mentioned limiting Froude number. There are two reasons for this conclusion: 1) the time interval for the accelerated run is the longest possible; 2) the nondimensional expressions for $\frac{V'}{V}$ or for the exciting forces are the largest possible.

The minimum factor γ necessary for this limiting Froude number will be larger than the above computed factor for $Fr = 0.142$.

Already the factor γ for $Fr = 0.142$ appears to be very large. Furthermore, it appears as a result of this estimate that the heading angle will grow rapidly during the accelerated run. This growth will be intensified because at the beginning of the accelerated run the rudder angle will be too small due to a relatively quiet run of the ship prior to the accelerated run.

The accelerated run lasts only a limited time interval. This time interval is estimated to be too short for broaching to occur. However, the heading angle will be larger at the end of this time interval than at the beginning. Broaching will occur if a run with a constant but increased speed follows the accelerated run. Although then the surging force is needed to overcome the increased ship's resistance, a swaying force also exists in this second time interval.

The following equations must be used for this second time interval instead of eq. 70:

$$-m_y\dot{\beta}+(m_x-Y_r)\dot{\psi}-Y_\beta\dot{\beta} = Y_\delta\gamma(\psi-\psi_o) - m_x\psi \frac{a}{F_r^2}\sqrt{\int_0^\infty r \frac{\omega^4}{g^2} f_{xx}^2\left(\frac{\lambda}{L}\right) d\omega} \tag{72}$$

$$n_z\ddot{\psi} - N_\beta\beta - N_r\dot{\psi} = N_\delta\gamma(\psi-\psi_o)$$

The following mimimum factor γ computed from these equations amounts to:

			for ship A	for ship C
for $\frac{u^2}{gL} = \frac{1}{2}$;	F_r=0.48;	γ min.	0.43	0.78
for $\frac{u^2}{gL} = \frac{1}{3}$;	F_r=0.47;	γ min.	0.31	0.55

This minimum factor γ appears to be too large for ship C and therefore it is concluded that broaching cannot be prevented for ship C in such seas.

CONCLUSIONS

The course keeping of a ship in a severe following or quartering sea is discussed only in a rough fashion. However, the following conclusions appear to be justified:

1. The surging and swaying exciting forces are more important for broaching than the yawing exciting moment.

2. Broaching probably will be more severe for a slow running than for a fast running ship, provided that the slow running ship still can be accelerated by the sea to the dominant wave speed (see Appendix).

3. Broaching is more probable for a ship course-unstable in calm water than for a ship course-stable in calm water.

4. The suspected connection between broaching and the nonlinear surging motion is confirmed by conclusion 2.

5. The criterion for an acceleration of the ship to the dominant wave speed can be used as an important criterion for broaching. This criterion will be more severe for a slow running ship than for a faster running ship.

6. The method developed in Part I can be used for determining this criterion.

The most interesting of these conclusions is the second one. To emphasize this conclusion, it is added that a ship running with the dominant wave speed in a severe following irregular sea is extremely hard to steer and steering control may be lost at times. The tendency to broach will always be present, but a severe yawing motion and tendency for broaching are not necessarily the same. Broaching is a more spontaneous motion and connected with an impetuous surging motion. Therefore it appears possible that the most severe broaching tendency will occur for a ship running with a speed slower than the dominant wave speed.

RECOMMENDATIONS

1. Model tests to validate the results predicted in Part I.
2. Determine the effect of hull form on the parameters used in Part I.
3. Determine the upper limit curve for broaching (see Appendix).
4. Determine the effect of hull form on broaching.
5. Investigate the rolling motion connected with broaching.

ACKNOWLEDGMENTS

The author wishes to thank the Davidson Laboratory, Stevens Institute of Technology for the opportunity to stay at their laboratory for some time and to carry on this investigation. He also wishes to thank Mr. Edward Numata and Dr. I. Robert Ehrlich for their help in the review of this paper.

REFERENCES

1. St. Denis, M. and Pierson, W. J., Jr.: "On the Motions of Ships in Confused Seas," SNAME Trans., Vol. 61, 1953.
2. Grim, O.: "Das Schiff in von Achtern Auflaufender See," Jahrbuch der Schiffbautechnischen Gesellschaft, 45 Band, 1951.
3. DuCane, P. and Goodrich. G. J.: "The Following Sea, Broaching and Surging," The Royal Institution of Naval Architects, Quarterly Transactions April 1962, Vol. 104, No. 2.
4. Jahnke, E. and Emde, F.: "Tables of Functions," Dover Publications, New York.
5. Davidson, K. S. M. and Schiff, L.: "Turning and Course Keeping Qualities," SNAME Trans., Vol. 54, 1946.
6. Schiff, L. and Gimprich, M.: "Automatic Steering of Ships by Automatic Control," SNAME Trans., Vol. 57, 1949.

NOMENCLATURE

a	coefficient for the average $\frac{1}{n}$ highest values
b	acceleration
$\bar{b}$	amplitude of acceleration in regular waves
C	coefficient for the Neumann spectrum
$F_r = \frac{V}{\sqrt{gL}}$	Froude number
F_{xx}, F_{yy}, F_{zz}	exciting forces
$f_{xx}(\frac{\lambda}{L})$, $f_{yy}(\frac{\lambda}{L})$, $f_{zz}(\frac{\lambda}{L})$	nondimensional coefficients for exciting forces in regular waves
g	gravitational acceleration
$h(x,t)$	displacement of the water surface in the vertical direction
$\bar{h}$	wave amplitude of a regular wave
H	draft
I_z	moment of inertia
L	length of the ship
M_{xx}, M_{yy}, I_{zz}	added masses and added moments of inertia
$m_x = \frac{m + M_{xx}}{\frac{\rho}{2} L^2 H}$; $m_y = \frac{m + M_{yy}}{\frac{\rho}{2} L^2 H}$	nondimensional total masses
m	mass
N	coefficient of damping force for the surging motion

N_β, N_r, N_δ	nondimensional hydrodynamic moment coefficients
$n_z = \frac{I_z + I_{zz}}{\frac{\rho}{2} L^4 H}$	nondimensional moment of inertia
$r(\omega)$	spectrum of a long-crested irregular sea
R	ship resistance
R_v	a constant value which corresponds to the effective resistance of a ship driven by a propeller running at constant speed
T	propeller thrust
t, τ, T	time or time intervals
u	wind speed
V	mean velocity for which calm water resistance = propeller thrust
x, y, z	coordinates
x_o	displacement of the surging motion
Y_β, Y_r, Y_δ	nondimensional hydrodynamic lateral force coefficients
ω	circle frequency referring to a fixed coordinate system
ω_e	freqency of encounter
ρ	density of water
λ	wave length of a regular wave
∇	displacement
ψ	heading angle from the direction of the regular or long-crested irregular sea
ψ_o	desired heading angle
δ	rudder angle
γ	rudder angle coefficient for a simple automatic steering
σ	arbitrary phase angle

APPENDIX

In the conclusions of the second section it is established that: "Broaching will be more severe for a slow running ship than for a fast running ship, provided that the slow running ship can be accelerated by the sea to the dominant wave speed."

Figure 14 is prepared to determine the limit for an acceleration to a dominant wave speed. The curves plotted in Fig. 14 are computed as described in Part 1 for the following increment of velocity:

$$\frac{bt}{\sqrt{gL}} = 0.40 - \frac{V}{\sqrt{gL}} \tag{73}$$

$\frac{\omega_o^2 L}{2g} = 3.1$ can be computed for this increment of velocity from eq. 53. Figure 11 then shows that a run with an increased speed is probable for an adequate time interval. Therefore, an acceleration of the ship--and after that a run with an increased speed--will be possible if the windspeed is larger than plotted in Fig. 14,

Figure 14 shows only the limit for an acceleration to a dominant wave speed. But it is concluded from the discussions in Part 2 that the limit for an acceleration to a dominant wave speed may also be the limit for broaching. Figure 14 also shows the acceleration b for the plotted wind speed.

From Fig. 14 it may be concluded that:

1. If a ship runs with a velocity less than the limit speed, it cannot be accelerated to a dominant wave speed and broaching cannot occur.
2. If a ship runs with a velocity near to the limit speed, broaching will not occur often, but when it does, it will be very severe.
3. If a ship runs with a velocity greater than the limit speed, broaching will occur more frequently but not as severely.

4. If a ship runs with a velocity much greater than the limit speed, it will frequently be accelerated to the dominant wave speed, but broaching may not occur because the increment of speed can only be small and because the hydrodynamic forces on the body and the rudder will be sufficient to maintain a steady course. There must exist, therefore, an upper velocity limit for broaching.

5. Broaching will not arise in a mild sea. Therefore the limit curve for a mild sea may not be meaningful.

The treatment in Part 2 is not sufficient for determining the limits mentioned in points 4 and 5 above. A further study is therefore necessary to determine the limits, the influence of the ship form on these limits and the effects discussed in this paper, and, further, to confirm these limits by model tests.

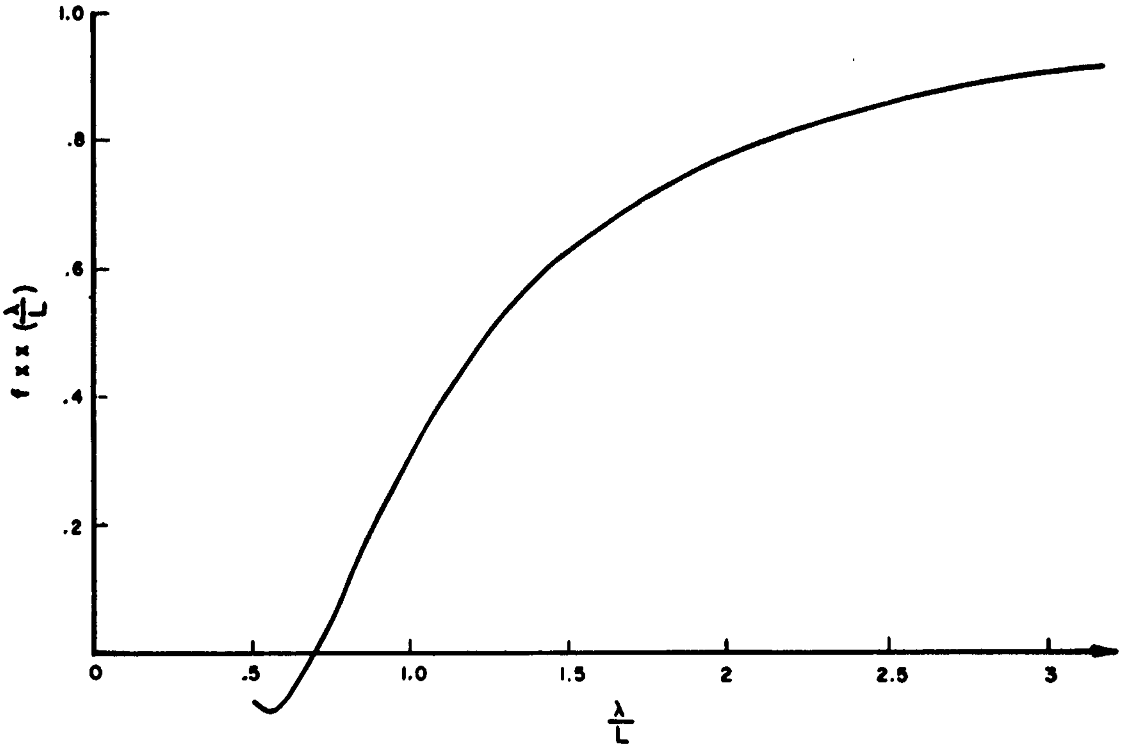


FIGURE 1. FUNCTION FOR THE SURGING EXCITING FORCE

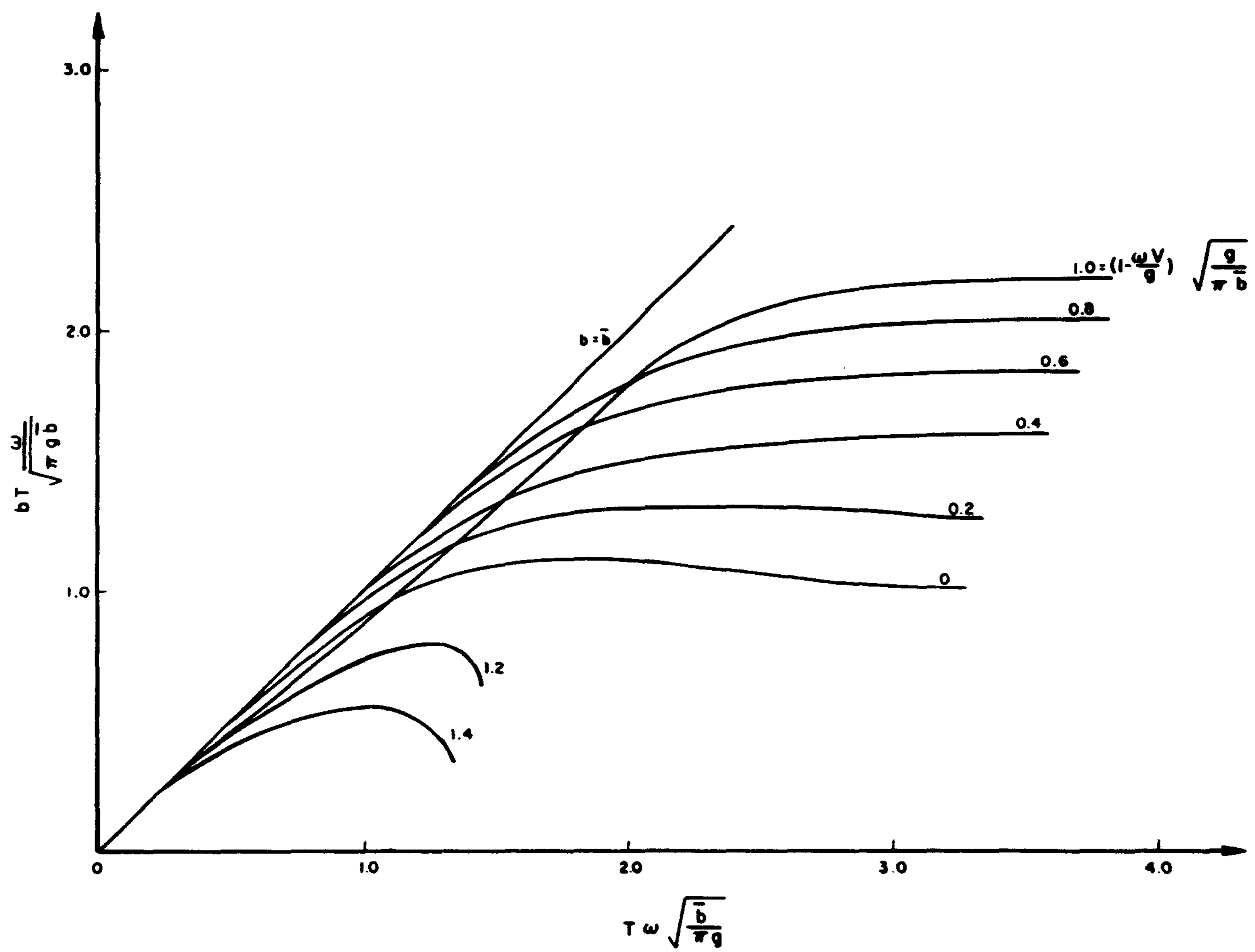


FIGURE 2. MAXIMUM INCREMENT OF VELOCITY PRODUCED BY REGULAR FOLLOWING WAVES

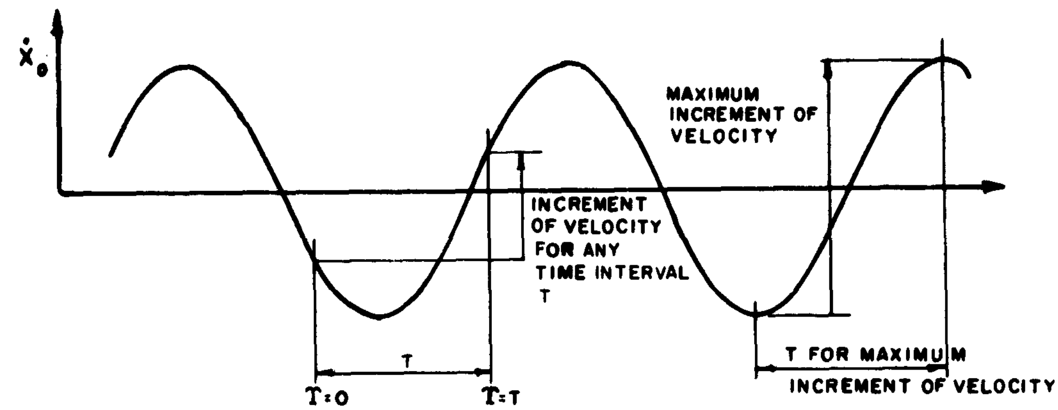


FIGURE 3. INCREMENT OF VELOCITY FOR A HARMONIC MOTION

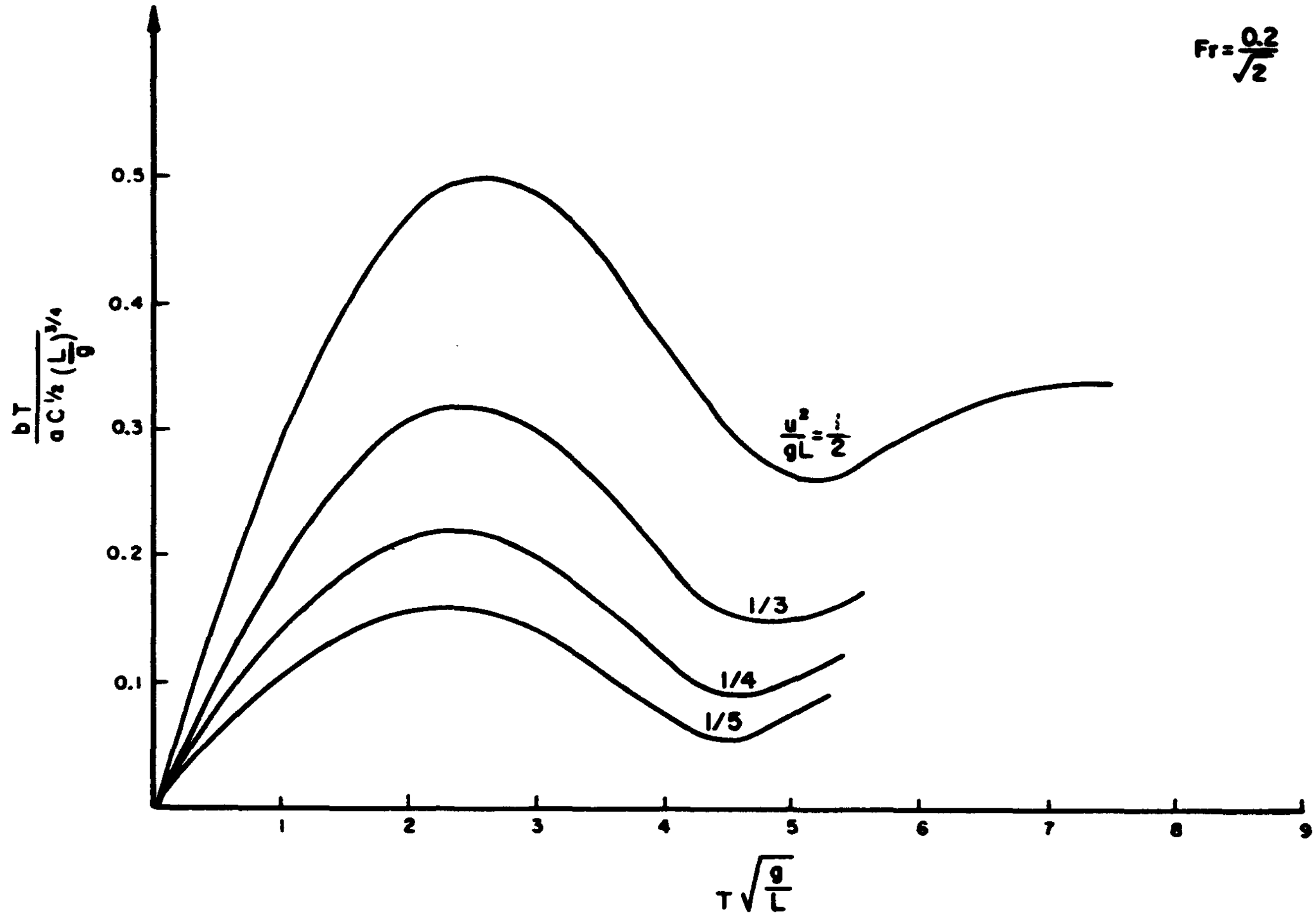


FIGURE 4. INCREMENT OF VELOCITY IN AN IRREGULAR FOLLOWING SEA ASSUMING LINEARIZED EQUATIONS OF MOTION

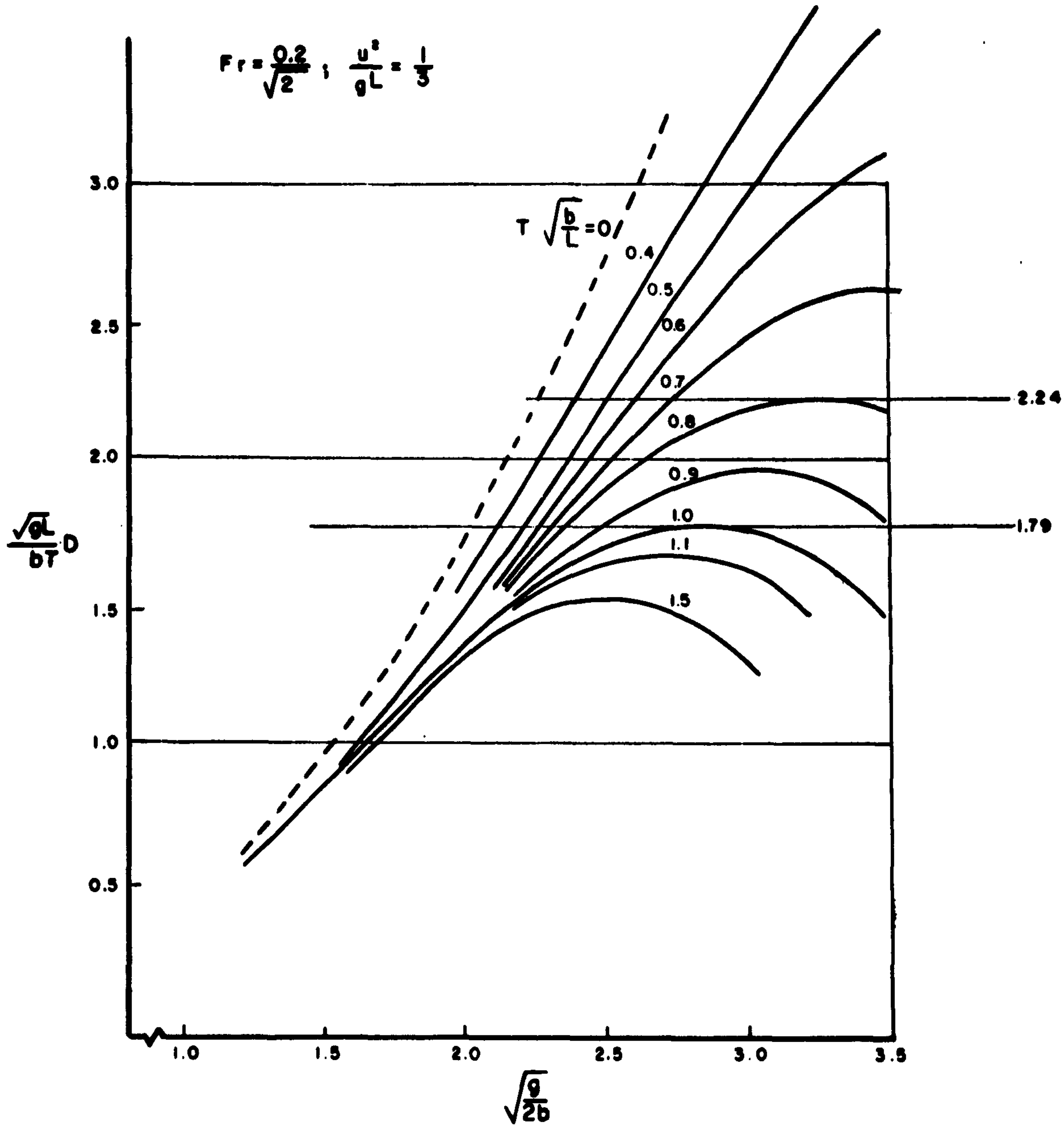


FIGURE 5. RESULTS FOR AN ASSUMED MOTION OF A CONSTANT ACCELERATION b IN A TIME INTERVAL T.

FIG. 6

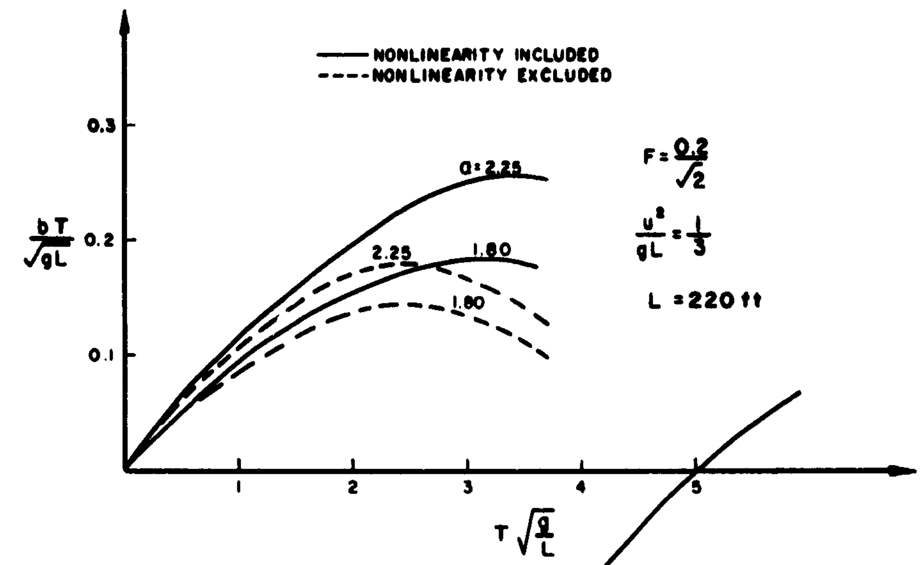


FIG. 7

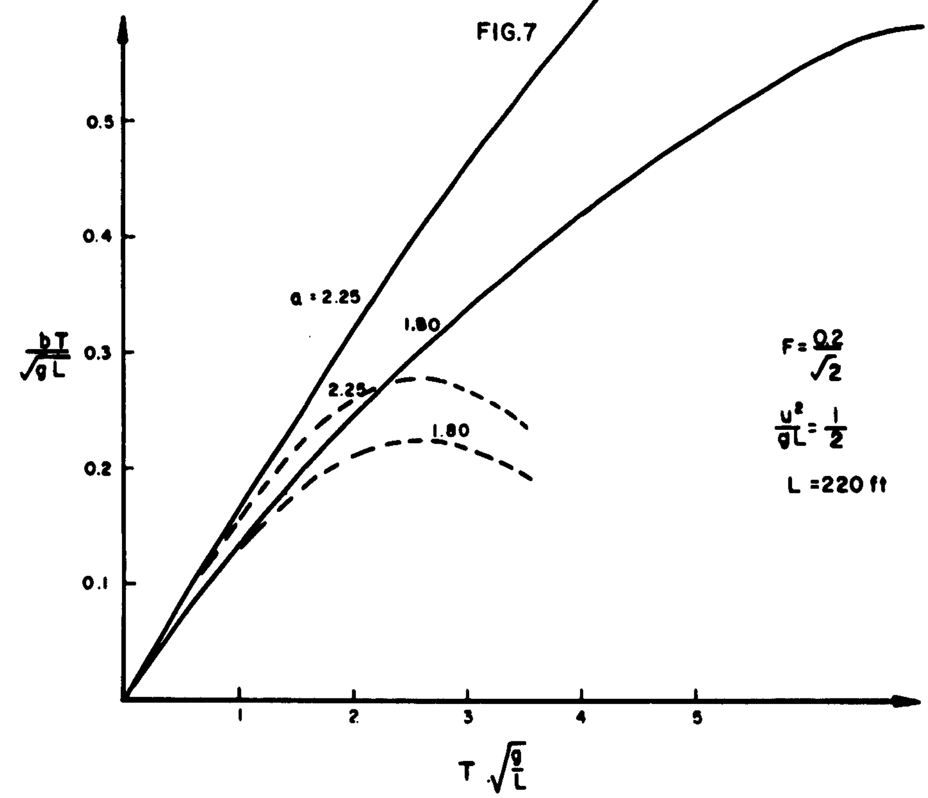


FIGURE 6 & 7. INCREMENT OF VELOCITY IN AN IRREGULAR FOLLOWING SEA

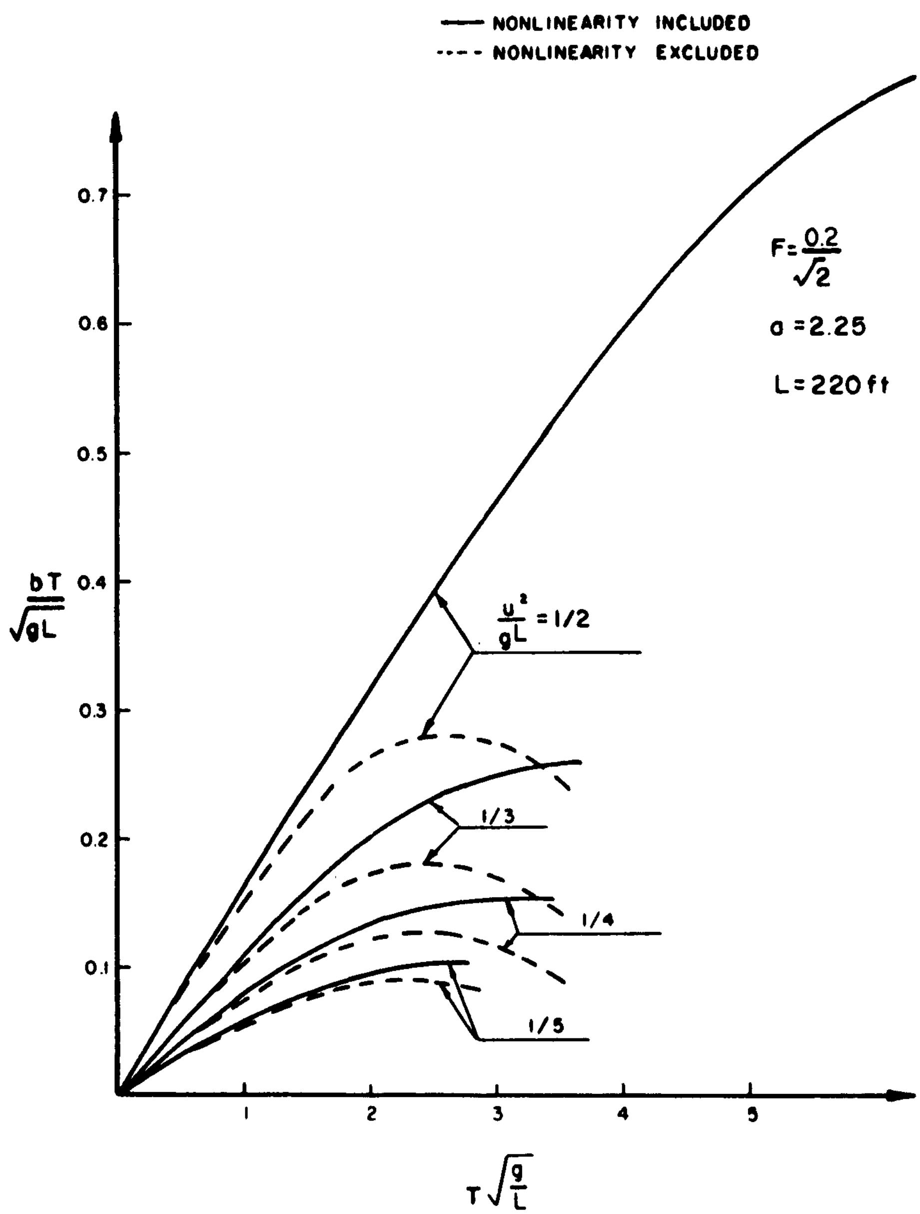


FIGURE 8. INCREMENT OF VELOCITY IN AN IRREGULAR FOLLOWING SEA

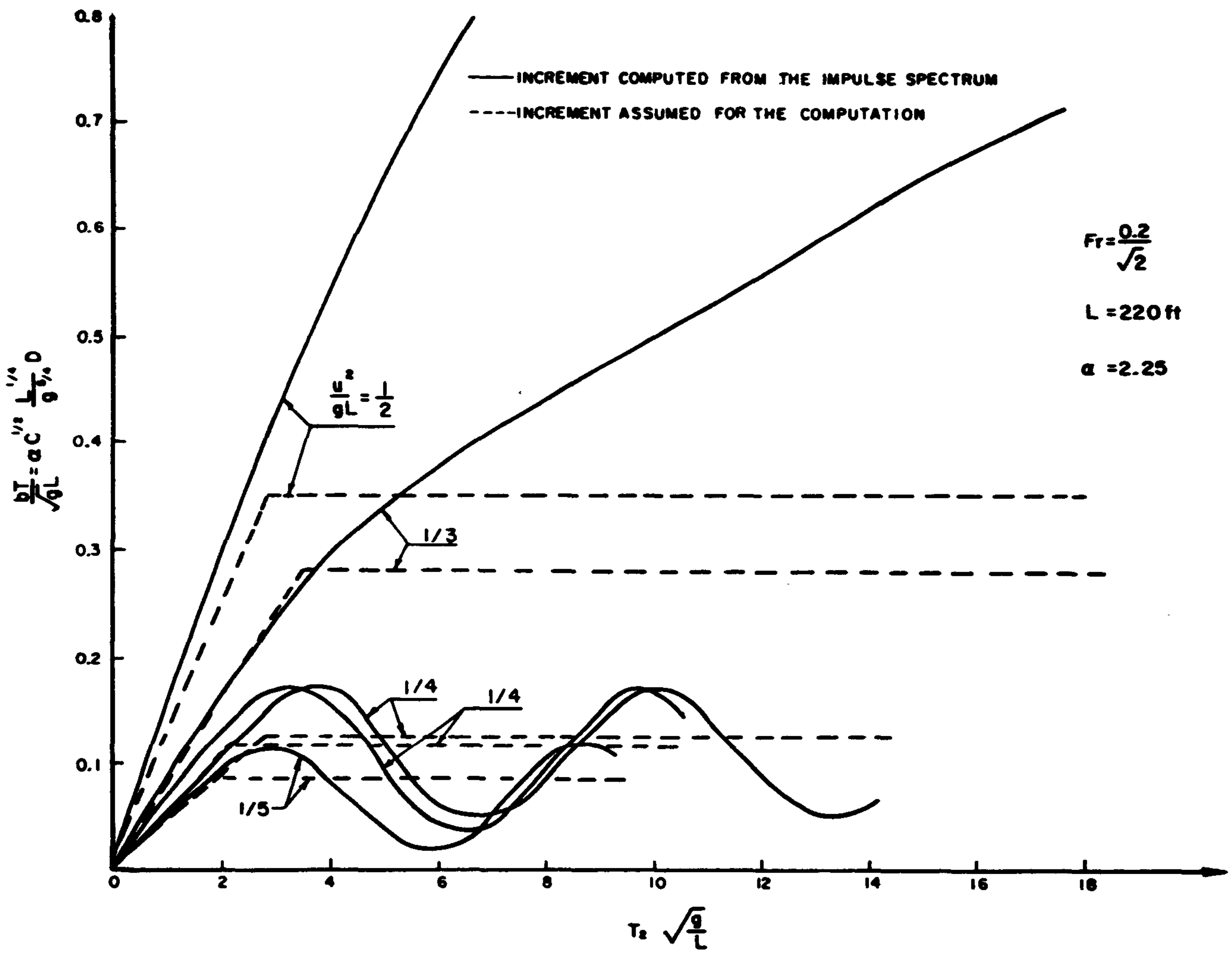


FIGURE 9. INCREMENT OF VELOCITY IN AN IRREGULAR FOLLOWING SEA WITH DAMPING NEGLECTED

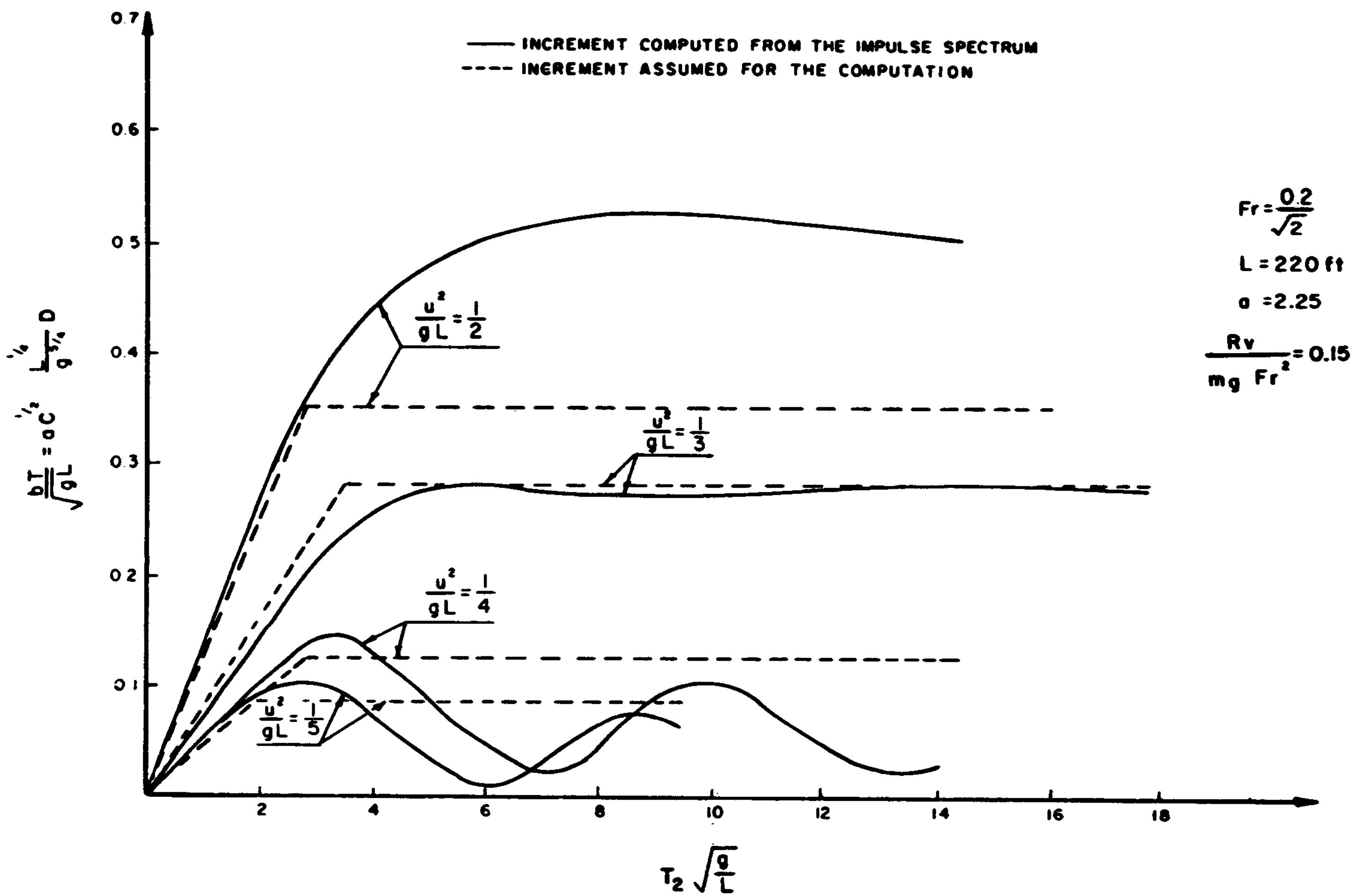


FIGURE 10. INCREMENT OF VELOCITY IN AN IRREGULAR FOLLOWING SEA WITH DAMPING INCLUDED

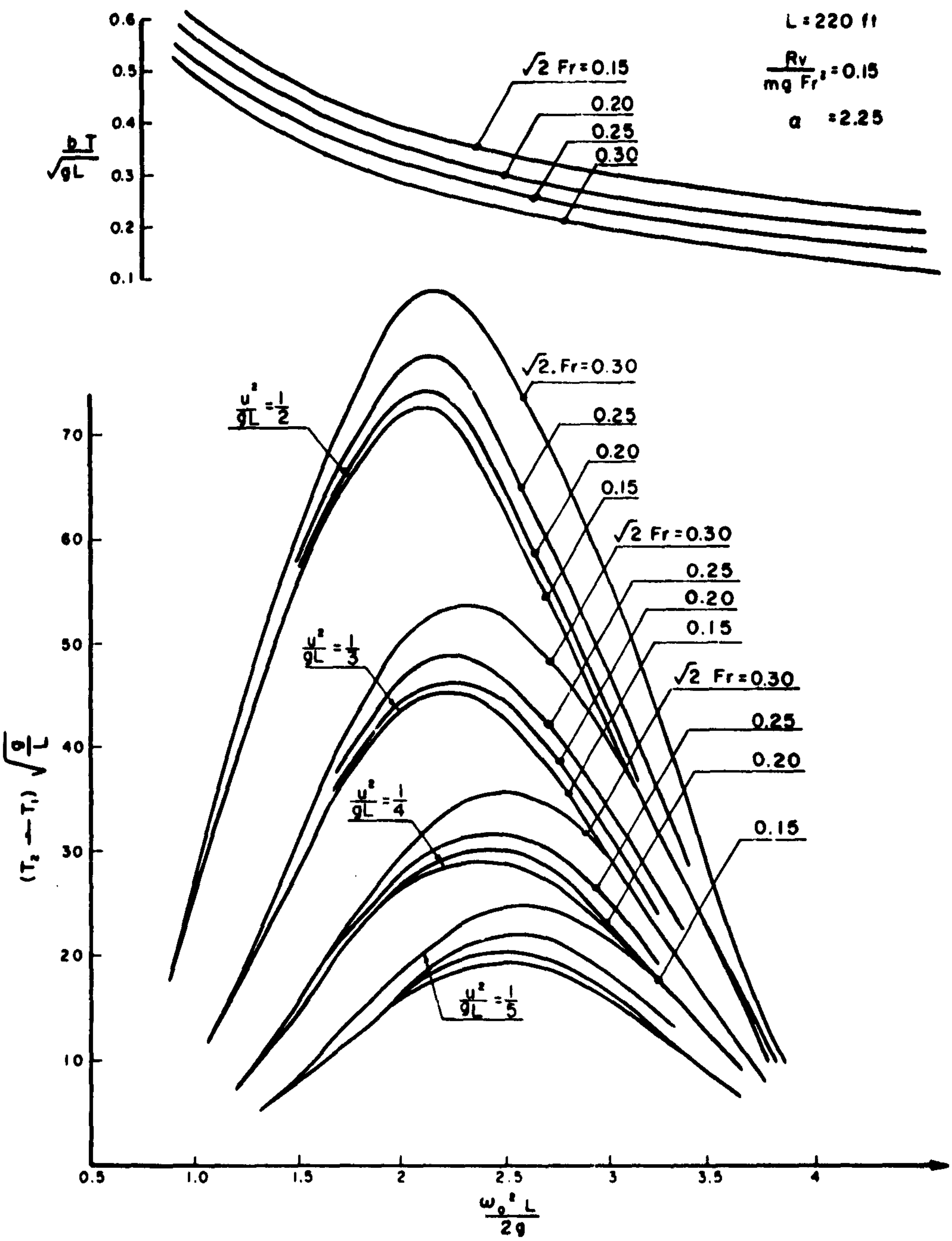


FIGURE 11. ESTIMATED TIME FOR A RUN OF THE SHIP AT INCREASED SPEED PLOTTED IN UPPER PART

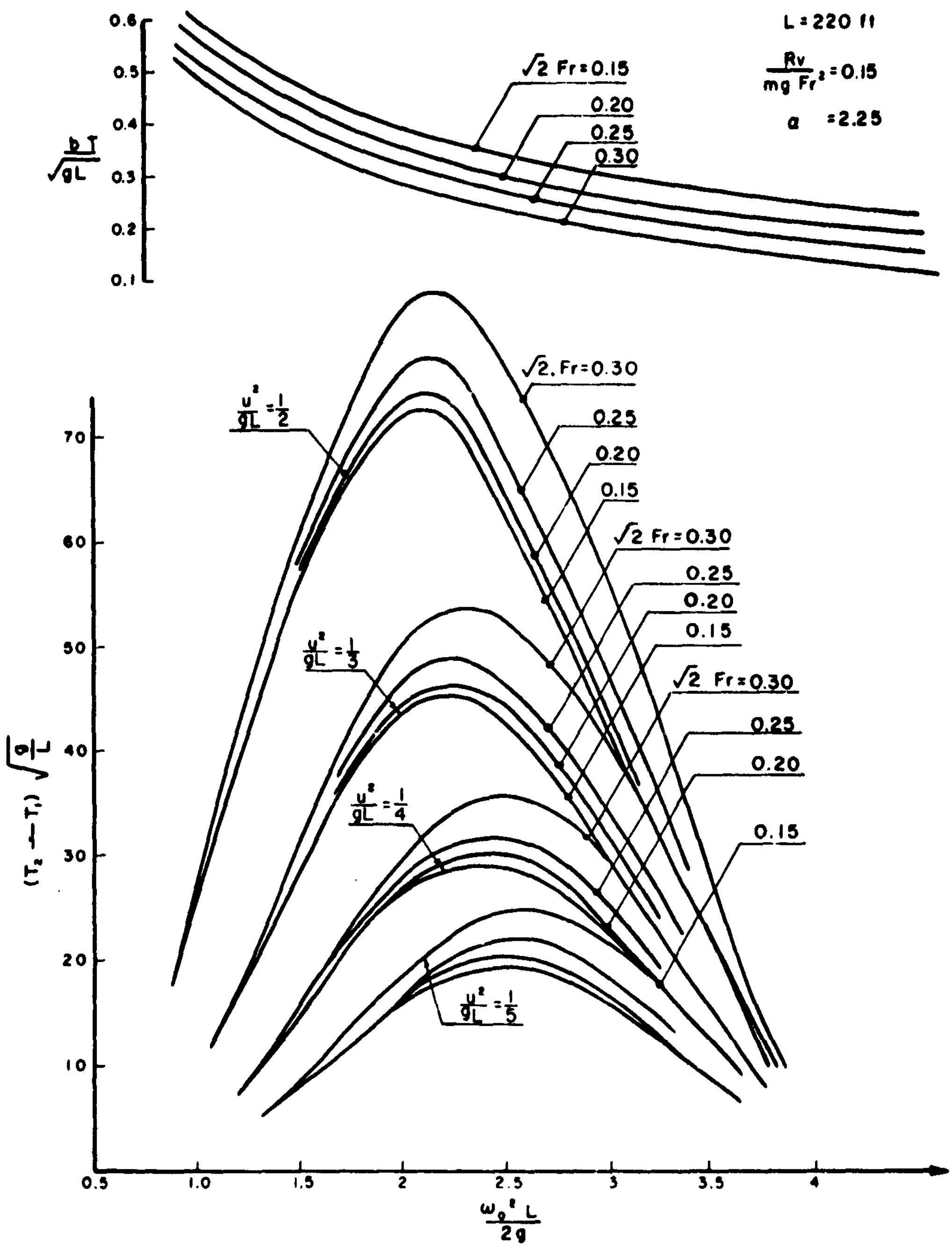


FIGURE 11. ESTIMATED TIME FOR A RUN OF THE SHIP AT INCREASED SPEED PLOTTED IN UPPER PART

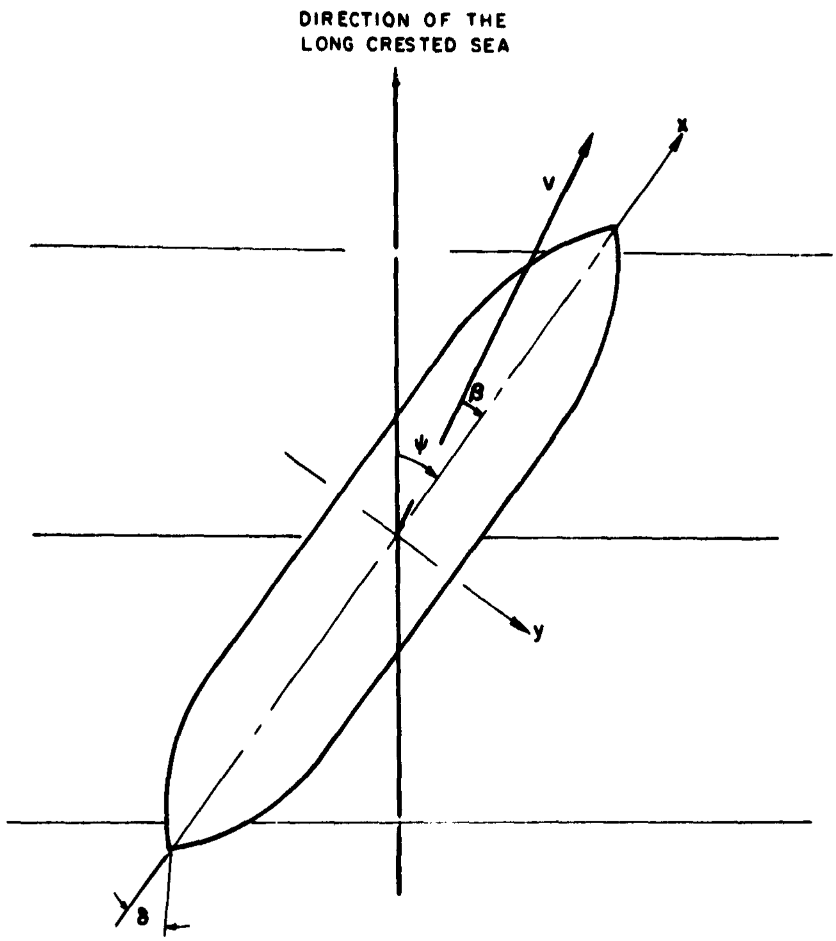


FIGURE 12. LOCATION OF SHIP IN WAVE

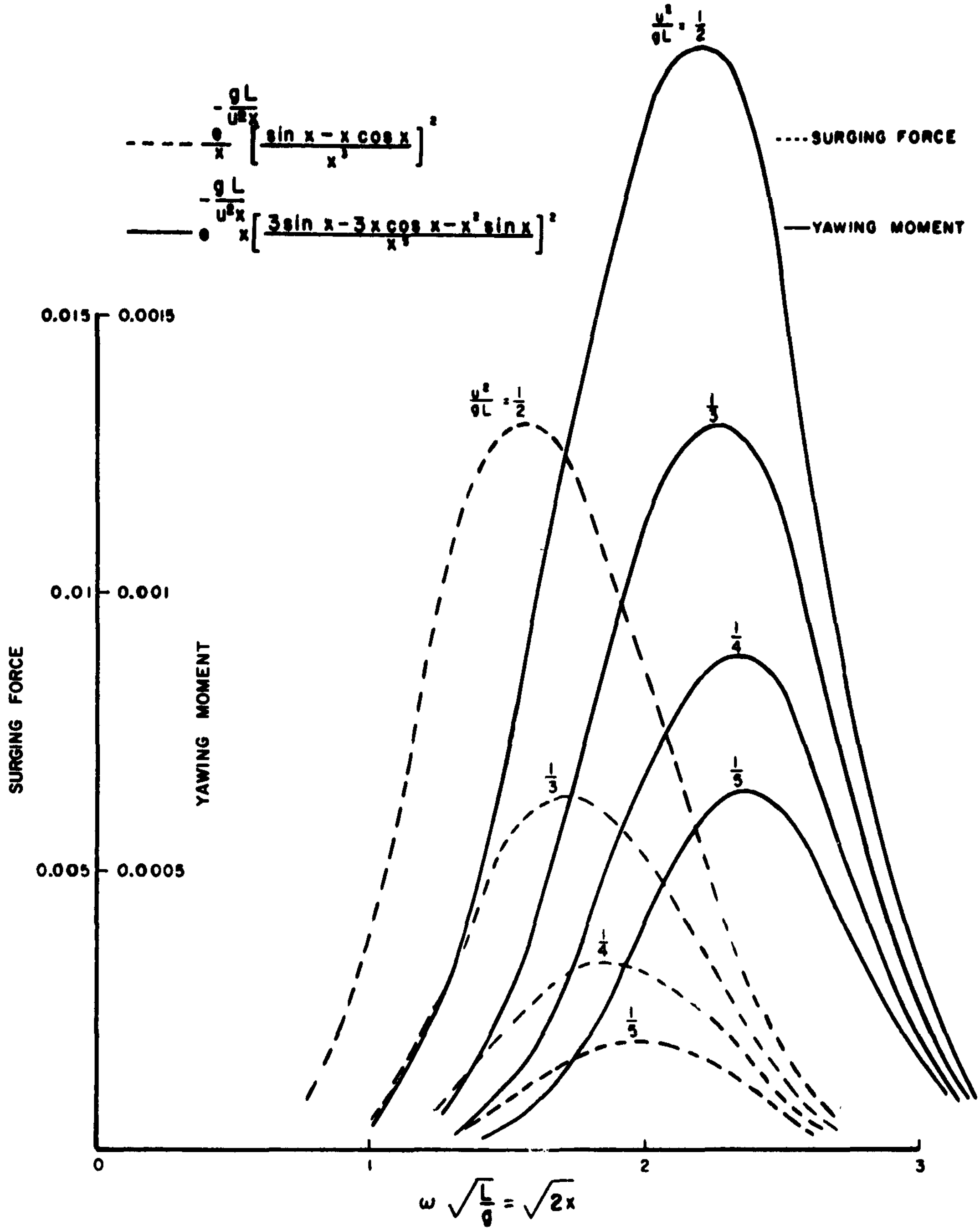


FIGURE 13. SPECTRA FOR SURGING EXCITING FORCES AND YAWING EXCITING MOMENTS

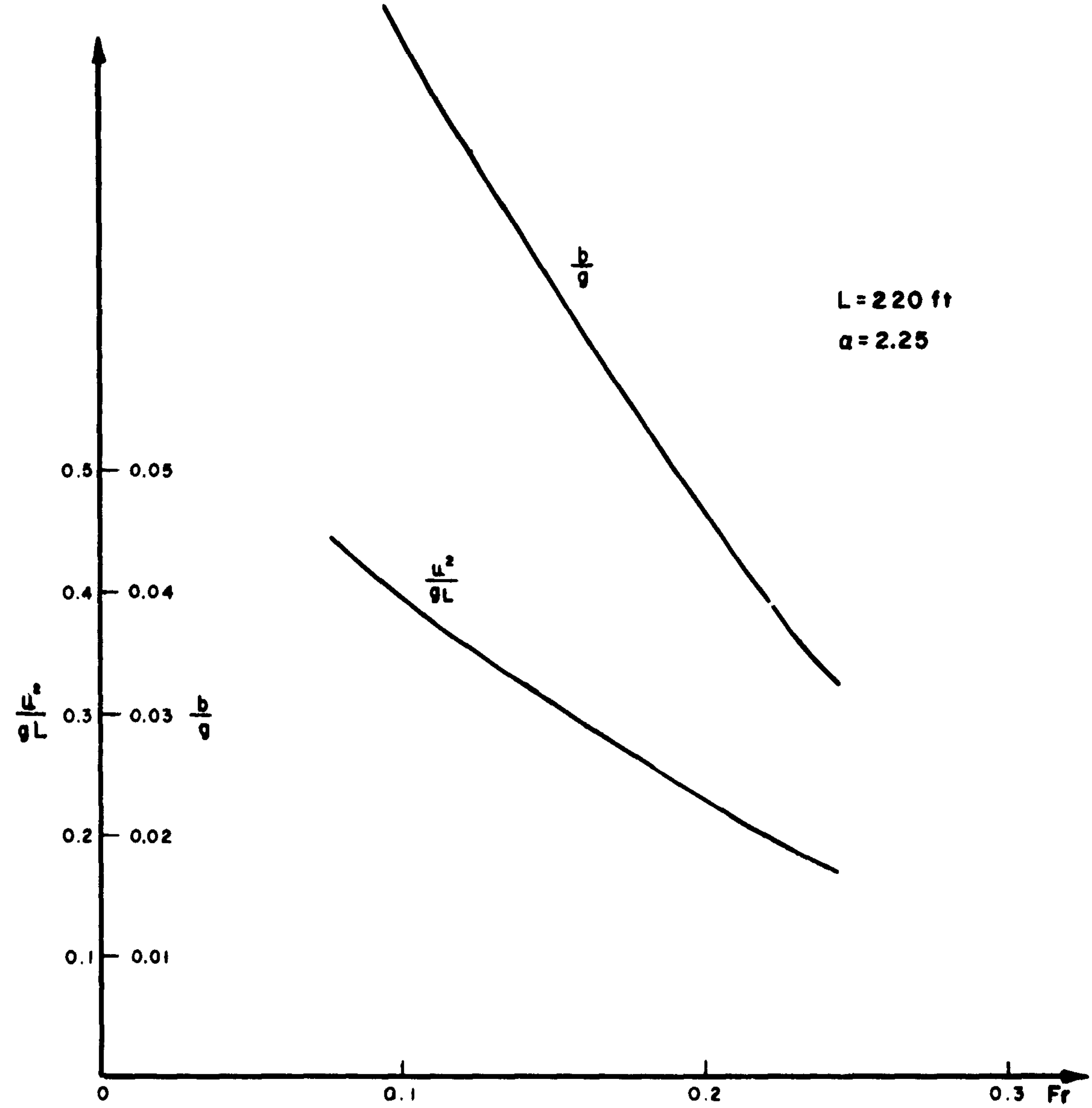


FIGURE 14. LOWEST WIND SPEED AND LOWEST ACCELERATION NECESSARY FOR AN ACCELERATION TO A SPEED OF Fr = 0.40

DISTRIBUTION LIST

Copies

6 Chief of Naval Research
Department of the Navy
Washington 25, D. C.
Attn: Codes 438 (3)
461 (1)
463 (1)
466 (1)

1 Commanding Officer
Office of Naval Research
Branch Office
495 Summer Street
Boston 10, Massachusetts

1 Commanding Officer
Office of Naval Research
Branch Office
207 West 24th Street
New York 11, New York

1 Commanding Officer
Office of Naval Research
Branch Office
1030 East Green Street
Pasadena, California

1 Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco 9, California

25 Commanding Officer
Office of Naval Research
Branch Office
Box 39, Navy # 100
Fleet Post Office
New York, New York

6 Director
Naval Research Laboratory
Washington 25, D. C.
Attn: Code 2027

5 Chief, Bureau of Naval Weapons
Department of the Navy
Washington 25, D. C.
Attn: Codes RUAW-4
RRRE
RAAD
RAAD-222
DIS-42

8 Chief, Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Codes 310
312
335
420
421
440
442
449

1 Chief, Bureau of Yards and Docks
Department of the Navy
Washington 25, D. C.
Attn: Code D-400

1 Commander
U. S. Naval Ordnance Test Station
China Lake, California
Attn: Code 753

2 Commander
U. S. Naval Ordnance Test Station
Pasadena Annex
3202 E. Foothill Blvd.
Pasadena 8, California
Attn: Code P-508

1 Commander
Planning Department
Portsmouth Naval Shipyard
Portsmouth, New Hampshire

1 Commander
Planning Department
Boston Naval Shipyard
Boston 29, Massachusetts

1 Commander
Planning Department
Pearl Harbor Naval Shipyard
Navy # 128, Fleet Post Office
San Francisco, California

1 Commander
Planning Department
San Francisco Naval Shipyard
San Francisco 24, California

1 Commander
Planning Department
Mare Island Naval Shipyard
Vallejo, California

DISTRIBUTION LIST

Copies

1 Director Eng. Sciences Div.
National Science Foundation
1951 Constitution Avenue, N.W.
Washington 25, D. C.

3 Director
National Bureau of Standards
Washington 25, D. C.
Attn: Fluid Mechanics Division
(Dr. G. B. Schubauer)
Dr. G. H. Keulegan
Dr. J. M. Franklin

10 Armed Services Technical
Information Agency
Arlington Hall Station
Arlington 12, Virginia

1 Office of Technical Services
Department of Commerce
Washington 25, D. C.

3 California Institute of Technology
Pasadena 4, California
Attn: Professor M. S. Plesset
Professor T. Y. Wu
Professor A. J. Acosta

1 University of California
Department of Engineering
Los Angeles 24, California
Attn: Dr. A. Powell

1 Director
Scripps Institute of Oceanography
University of California
La Jolla, California

1 Professor M. L. Albertson
Department of Civil Engineering
Colorado A&M College
Fort Collins, Colorado

1 Professor J. E. Cermak
Department of Civil Engineering
Colorado State University
Fort Collins, Colorado

1 Professor W. R. Sears
Graduate School of Aeronautical
Engineering
Cornell University
Ithaca, New York

2 State University of Iowa
Iowa Institute of Hydraulic Research
Iowa City, Iowa
Attn: Dr. H. Rouse
Dr. L. Landweber

2 Harvard University
Cambridge 38, Massachusetts
Attn: Professor G. Birkhoff
(Dept. of Mathematics)
Professor G. F. Carrier
(Dept. of Mathematics)

2 Massachusetts Institute of Technology
Cambridge 39, Massachusetts
Attn: Department of Naval Architecture and Marine Eng.
Professor A. T. Ippen

3 University of Michigan
Ann Arbor, Michigan
Attn: Professor R. B. Couch
(Dept. of Naval Arch.)
Professor W. W. Willmarth
(Aero. Engrg. Department)
Professor M. S. Uberoi
(Aero. Engrg. Department)

3 Dr. L. G. Straub, Director
St. Anthony Falls Hydraulic Lab.
University of Minnesota
Minneapolis 14, Minnesota
Attn: Mr. J. N. Wetzel
Prof. B. Silberman

1 Professor J. J. Foody
Engineering Department
New York State Univ. Maritime College
Fort Schulyer, New York

2 New York University
Inst. of Mathematical Sciences
25 Waverly Place
New York 3, N. Y.
Attn: Prof. J. Keller
Prof. J. J. Stoker

3 The Johns Hopkins University
Department of Mechanical Engineering
Baltimore 18, Maryland
Attn: Prof. S. Corrsin (1)
Prof. O. M. Phillips (2)

DISTRIBUTION LIST

Copies

1 Commander
Planning Department
New York Naval Shipyard
Brooklyn 1, New York

1 Commander
Planning Department
Puget Sound Naval Shipyard
Bremerton, Washington

1 Commander
Planning Department
Philadelphia Naval Shipyard
U. S. Naval Base
Philadelphia 12, Pennsylvania

1 Commander
Planning Department
Norfolk Naval Shipyard
Portsmouth, Virginia

1 Commander
Planning Department
Charleston Naval Shipyard
U. S. Naval Base
Charleston, South Carolina

1 Commander
Planning Department
Long Beach Naval Shipyard
Long Beach 2, California

1 Commander
Planning Department
U. S. Naval Weapons Laboratory
Dahlgren, Virginia

1 Commander
U. S. Naval Ordnance Laboratory
White Oak, Maryland

1 Dr. A. V. Hershey
Computation and Exterior Ballistics Laboratory
U. S. Naval Weapons Laboratory
Dahlgren, Virginia

1 Superintendent
U. S. Naval Academy
Annapolis, Maryland
Attn: Library

1 Superintendent
U. S. Naval Postgraduate School
Monterey, California

1 Commandant
U. S. Coast Guard
1300 E. Street, N. W.
Washington, D. C.

1 Secretary Ship Structure Committee
U. S. Coast Guard Headquarters
1300 E. Street, N. W.
Washington, D. C.

1 Commander
Military Sea Transportation Service
Department of the Navy
Washington 25, D. C.

2 U. S. Maritime Administration
GAO Building
441 G Street, N. W.
Washington, D. C.
Attn: Division of Ship Design
Division of Research

1 Superintendent
U. S. Merchant Marine Academy
Kings Point, Long Island, New York
Attn: Capt. L. S. McCready (Dept. of Engineering)

1 Commanding Officer and Director
U. S. Navy Mine Defense Laboratory
Panama City, Florida

1 Commanding Officer
NROTC and Naval Administrative Unit
Massachusetts Inst. of Technology
Cambridge 39, Massachusetts

1 U. S. Army Transportation Research and Development Command
Fort Eustis, Virginia
Attn: Marine Transport Division

1 Mr. J. B. Parkinson
National Aeronautics and Space Administration
1512 H Street, N. W.
Washington 25, D. C.

2 Director
Langley Research Center
Langley Station
Hampton, Virginia
Attn: Mr. I. E. Garrick
Mr. D. J. Marten

DISTRIBUTION LIST

Copies

2 Dr. G. F. Wislicenus
Ordnance Research Laboratory
Pennsylvania State University
University Park, Penna.
Attn: Dr. M. Sevik

1 Professor R. C. DiPrima
Department of Mathematics
Rensselaer Polytechnic Institute
Troy, New York

2 Webb Institute of Naval Arch.
Crescent Beach Road
Glen Cove, New York
Attn: Professor E. V. Lewis
Technical Library

1 Director
Woods Hole Oceanographic Institute
Woods Hole, Massachusetts

1 Executive Director
Air Force Office of Scientific Research
Washington 25, D. C.
Attn: Mechanics Branch

1 Commander
Wright Air Development Div.
Aircraft Laboratory
Wright-Patterson Air Force Base
Ohio
Attn: Mr. W. Mykytow, Dynamics

2 Cornell Aeronautical Laboratory
4455 Genesee Street
Buffalo, New York
Attn: Mr. W. Targoff
Mr. R. White

3 Mass. Institute of Technology
Fluid Dynamics Research Lab.
Cambridge 39, Massachusetts
Attn: Prof. H. Ashley
Prof. M. Landahl
Prof. J. Dugundji

1 Dr. S. F. Hoerner
148 Busteed Drive
Midland Park, New Jersey

1 Boeing Airplane Company
Seattle Division
Seattle, Washington
Attn: Mr. M. J. Turner

1 Electric Boat Division
General Dynamics Corporation
Groton, Connecticut
Attn: Mr. Robert McCandliss

1 General Applied Sciences Laboratories, Inc.
Merrick and Stewart Aves.
Westbury, L. I., N. Y.

1 Gibbs and Cox, Inc.
21 West Street
New York, N. Y.

3 Grumman Aircraft Eng. Corp.
Bethpage, L. I., N. Y.
Attn: Mr. E. Baird
Mr. E. Bower
Mr. W. P. Carl

1 Lockheed Aircraft Corporation
Missiles and Space Div.
Palo Alto, California
Attn: R. W. Kermeen

1 Midwest Research Institute
425 Volker Blvd.
Kansas City 10, Missouri
Attn: Mr. Zeydel

3 Director, Dept. of Mech. Sciences
Southwest Research Institute
8500 Culebra Road
San Antonio 6, Texas
Attn: Dr. H. N. Abramson
Mr. G. Ransleben
Editor, Applied Mechanics Review

2 Convair
A Division of General Dynamics
San Diego, California
Attn: Mr. R. H. Oversmith
Mr. H. T. Brooke

1 Hughes Tool Company
Aircraft Division
Culver City, California
Attn: Mr. M. S. Harned

1 Hydronautics, Incorporated
Pindell School Road
Howard County
Laurel, Maryland
Attn: Mr. Phillip Eisenberg

DISTRIBUTION LIST

Copies

1 Rand Development Corporation
13600 Deise Avenue
Cleveland 10, Ohio
Attn: Dr. A. S. Iberall

1 U. S. Rubber Company
Research and Development Dept.
Wayne, New Jersey
Attn: Mr. L. M. White

1 Technical Research Group, Inc.
2 Aerial Way
Syosset, L. I., N. Y.
Attn: Dr. Jack Kotik

1 AVCO Corporation
Lycoming Division
1701 K Street, N. W., Apt. 904
Washington, D. C.
Attn: Mr. T. A. Duncan

1 Mr. J. G. Baker
Baker Manufacturing Company
Evansville, Wisconsin

1 Curtiss-Wright Corp. Research Div.
Turbomachinery Division
Quehanna, Pennsylvania
Attn: Mr. George Pedersen

1 Dr. Blaine R. Parkin
AiResearch Manufacturing Corp.
9851-9951 Sepulveda Blvd.
Los Angeles 45, California

1 The Boeing Company
Aero-Soace Division
Seattle 24, Washington
Attn: Mr. R. E. Bateman
(Internal Mail Station 46-74)

1 Lockheed Aircraft Corporation
California Division
Hydrodynamics Research
Burbank, California
Attn: Mr. Bill East

1 Mass. Institute of Technology
Dept. of Naval Architecture and
Marine Engineering
Cambridge 39, Massachusetts
Attn: Prof. M. A. Abkowitz

2 Stanford University
Dept. of Civil Engineering
Stanford, California
Attn: Dr. Byrne Perry
Dr. E. Y. Hsu

1 Dr. Hirsh Cohen
IBM Research Center
P. O. Box 218
Yorktown Heights, New York

1 Mr. David Wellinger
Hydrofoil Projects
Radio Corporation of America
Burlington, Massachusetts

1 Food Machinery Corporation
P. O. Box 367
San Jose, California
Attn: Mr. G. Tedrew

1 Dr. T. R. Goodman
Oceanics, Inc.
Technical Industrial Park
Plainview, L. I., N. Y.

1 Professor Brunelle
Dept. of Aeronautical Engineering
Princeton University
Princeton, New Jersey

1 Commanding Officer
Office of Naval Research Br Ofc
86 East Randolph Street
Chicago 1, Illinois

1 The Rand Corporation
1700 Main Street
Santa Monica, California
Attn: Technical Library

1 National Research Council
Montreal Road
Ottawa 2, Canada
Attn: Mr. E. S. Turner

DISTRIBUTION LIST

Copies

16 Commanding Officer and Director
David Taylor Model Basin
Washington 7, D. C.
Attn: Codes 108
142
500
513
520
525
526
526A
530
533
580
585
589
591
591A
700

1 Hamburgische Schiffbau-Versuchsanstalt
Bramfelder Strasse 164
Hamburg 33, Germany
Attn: Dr. H. W. Lerbs

1 Institut für Schiffbau der Universitat Hamburg
Berliner Tor 21
Hamburg 1, Germany
Attn: Dr. O. Grim

1 Trans. Technical Research Inst.
1-1057, Mejiro-Cho, Toshima-Ku
Tokyo, Japan

1 Max-Planck Institut für Strömungsforschung
Bottingerstrasse 6/8
Gottingen, Germany
Attn: Dr. H. Reichardt

1 Hydro-og Aerodynamisk Laboratorium
Lyngby, Denmark
Attn: Prof. Carl Prohaska

3 Versuchsanstalt für Wasserbau und Schiffbau
Schleuseninsel im Tiergarten
Berlin, Germany
Attn: Dr. S. Schuster, Director
Dr. H. Schwanecke
Dr. Grosse

1 Technische Hogeschool
Institut voor Toegepaste Wiskunde
Julianalaan 132
Delft, Netherlands
Attn: Prof. R. Timman

1 Bureau D'Analyse et de Recherche Appliquees
47 Avenue Victor Cresson
Issy-Les-Moulineaux
Seine, France
Attn: Prof. Siestrunck

1 Netherlands Ship Model Basin
Wageningen, The Netherlands
Attn: Dr. Ir. J. D. van Manen

2 National Physical Laboratory
Teddington, Middlesex, England
Attn: Mr. A. Silverlead
Head, Aerodynamics Div.

2 Head, Aerodynamics Department
Royal Aircraft Establishment
Farnborough, Hants, England
Attn: Mr. M. O. W. Wolfe

1 Skipsmodelltanken
Trondheim, Norway
Attn: Professor J. K. Lunde

1 Mr. C. Wigley
Flat 102
6-9 Charterhouse Square
London E.C.1
England

Davidson Laboratory Report No. 929 UNCLASSIFIED

SURGING MOTION AND BROACHING TENDENCIES IN A SEVERE IRREGULAR SEA

by Dr. Otto Grim, November 1962

Nonlinearities are very significant for the surging motion of a ship in a following sea. Results are presently available only for the surging motion in regular following waves. The law of superposition cannot be used to treat the nonlinear surging motion in an irregular sea. Therefore, it is not possible to extend the results obtained in regular waves to predict the behavior in an irregular sea.

A method to overcome these difficulties to investigate nonlinear surging motion in an irregular sea is presented in Part 1 of this report. Using this method, results are obtained for nonlinear surging motions; in particular, solutions are given for the acceleration of a ship to an increased speed and for the subsequent run at that increased speed.

Based on the results obtained in Part 1, the problem of broaching is discussed in Part 2. A connection between broaching and nonlinear surging motion was presumed in previous discussions about broaching. This connection is confirmed and, furthermore, nonlinear surging motion is found to be the most important pre-condition for broaching.

Davidson Laboratory Report No. 929 UNCLASSIFIED

SURGING MOTION AND BROACHING TENDENCIES IN A SEVERE IRREGULAR SEA

by Dr. Otto Grim, November 1962

Nonlinearities are very significant for the surging motion of a ship in a following sea. Results are presently available only for the surging motion in regular following waves. The law of superposition cannot be used to treat the nonlinear surging motion in an irregular sea. Therefore, it is not possible to extend the results obtained in regular waves to predict the behavior in an irregular sea.

A method to overcome these difficulties to investigate nonlinear surging motion in an irregular sea is presented in Part 1 of this report. Using this method, results are obtained for nonlinear surging motions; in particular, solutions are given for the acceleration of a ship to an increased speed and for the subsequent run at that increased speed.

Based on the results obtained in Part 1, the problem of broaching is discussed in Part 2. A connection between broaching and nonlinear surging motion was presumed in previous discussions about broaching. This connection is confirmed and, furthermore, nonlinear surging motion is found to be the most important pre-condition for broaching.

Davidson Laboratory Report No. 929 UNCLASSIFIED

SURGING MOTION AND BROACHING TENDENCIES IN A SEVERE IRREGULAR SEA

by Dr. Otto Grim, November 1962

Nonlinearities are very significant for the surging motion of a ship in a following sea. Results are presently available only for the surging motion in regular following waves. The law of superposition cannot be used to treat the nonlinear surging motion in an irregular sea. Therefore, it is not possible to extend the results obtained in regular waves to predict the behavior in an irregular sea.

A method to overcome these difficulties to investigate nonlinear surging motion in an irregular sea is presented in Part 1 of this report. Using this method, results are obtained for nonlinear surging motions; in particular, solutions are given for the acceleration of a ship to an increased speed and for the subsequent run at that increased speed.

Based on the results obtained in Part 1, the problem of broaching is discussed in Part 2. A connection between broaching and nonlinear surging motion was presumed in previous discussions about broaching. This connection is confirmed and, furthermore, nonlinear surging motion is found to be the most important pre-condition for broaching.

Davidson Laboratory Report No. 929 UNCLASSIFIED

SURGING MOTION AND BROACHING TENDENCIES IN A SEVERE IRREGULAR SEA

by Dr. Otto Grim, November 1962

Nonlinearities are very significant for the surging motion of a ship in a following sea. Results are presently available only for the surging motion in regular following waves. The law of superposition cannot be used to treat the nonlinear surging motion in an irregular sea. Therefore, it is not possible to extend the results obtained in regular waves to predict the behavior in an irregular sea.

A method to overcome these difficulties to investigate nonlinear surging motion in an irregular sea is presented in Part 1 of this report. Using this method, results are obtained for nonlinear surging motions; in particular, solutions are given for the acceleration of a ship to an increased speed and for the subsequent run at that increased speed.

Based on the results obtained in Part 1, the problem of broaching is discussed in Part 2. A connection between broaching and nonlinear surging motion was presumed in previous discussions about broaching. This connection is confirmed and, furthermore, nonlinear surging motion is found to be the most important pre-condition for broaching.